MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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F. W. DYSON, Esq., M.A., LL.D., F.R.S., PRESIDENT, in the Chair.

Harry Hinsley Burgham, 34 Aldersgate Street, E.C.;
Rev. William Wyatt Holdgate, M.A., Sutton Valence School, near Maidstone, Kent; and
Harry Russell Maynard, B.A., Kingswood, Antrim Street, Haverstock Hill, N.W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Alexander Granville Blake, Governor's House, Chelmsford, Essex (proposed by W. H. M. Christie);
Arthur Lock, Navigating Officer, s.s. *Rohilla*, 268 Newport Road, Cardiff (proposed by J. Netherclift Jutsum);
Algernon Montagu Newbegin, Inverurie, Talbot Road, Carshalton, Surrey (proposed by G. J. Newbegin); and
Thomas Nathaniel Orchard, Doctor of Medicine, Ashfield House, Seedley Road, Pendleton, Manchester (proposed by Walter Sidgreaves).

Fifty-six presents were announced as having been received since the last meeting, including, amongst others:—

Twelve transparencies from celestial photographs taken at the Royal Observatory, Cape of Good Hope, presented by the

Observatory; and 20 charts of the Astrographic Chart of the Heavens, presented by the Observatory, San Fernando.

A. R. Hinks, Solar Parallax Papers, 1-9, presented by the Author.

Eclipse Observations.

The Joint Permanent Eclipse Committee of the Royal Society and Royal Astronomical Society will, in the autumn of this year, take into consideration the question of the observation of the eclipse of 1914 Aug. 21.

Intending observers are requested to send their proposed programmes of work to the Secretary before Oct. 1st next.

British observers who wish to apply for financial assistance must, at the same time, furnish detailed estimates of the expenditure they wish covered, for submission to the Government Grant Committee in the event of their programmes being approved by the Eclipse Committee.

Attention is directed to the suggested lines of work printed in *Monthly Notices*, vol. lxxii. page 90 (Dec. 1911).

E. H. Hills, *Sec. J.P.E.C.*

The Spectrum of the Sun's Limb during the Partial Eclipse of the Sun, 1912 April 16–17. By H. F. Newall, M.A., D.Sc., F.R.S.

The partial eclipse of the Sun was visible under very favourable meteorological conditions at Cambridge on 1912 April 16–17. According to the data of the *Nautical Almanac* the magnitude of the eclipse was 0·906 (Sun's diameter = 1), and at the greatest phase the atmospheric glare in the neighbourhood of the Sun's limb would be reduced to about 12 per cent. of what it would be with uneclipsed Sun. Under these circumstances it was thought that the bright lines of the flash spectrum might be photographed with useful effect.

The spectrum of the Sun's limb was accordingly photographed with the Littrow spectrograph. An image of the crescent tip was thrown on the slit by means of the double cœlostat and the 12-inch lens, which has a focal length of 60 feet, and photographs were taken in the spectrum of the first order of the 6-inch Michelson grating, giving a linear dispersion of 1 mm. to 4·02 tenth-metres in the spectral region between wave-lengths 4040 and 4400.

The following table gives the essential details of the photographs taken:—

No.	Time.			Position Angle from N. Point through E.	Exposure.	Remarks.
		h	m			
3	Apr. 16	23	0	...	5 sec.	Solar spectrum at 3 different focus readings; that with best definition serves as reference spectrum.
4	,,	23	47	148°	10 ,,	$H\delta$ and $H\gamma$ bright; continuous spectrum very faint.
5	Apr. 17	0	5	97	10 ,,	ditto ditto.
7	,,	0	8	64·1	10 ,,	Shows between 30 and 40 bright lines, many exceedingly narrow. Continuous spectrum very faint.
8	,,	0	13	66·4	20 ,,	Modified solar spectrum showing between 20 and 30 bright lines, and many of the Fraunhofer lines either obliterated or much weakened.

No. 8 shows that the atmospheric glare was still considerable, even at the greatest phase of the eclipse, which occurred at $0^h\ 12^m$. For in an exposure of 20 seconds the Fraunhofer spectrum asserts itself with an intensity *comparable* with, though fainter than, the spectrum which the bright solar surface gives in one quarter of the exposure. The image of the bright limb of the Sun had been set about $0^{mm}\cdot25$ ($=2''\cdot8$) from the actual slit, in the hope that the direct photospheric light would be kept out of the slit in spite of atmospheric tremor.

Nos. 3, 7, and 8 have been measured for a preliminary survey of the peculiarities exhibited.

It is clear that No. 7 is a "flash" spectrum of what may be described as the upper stratum of the reversing layer. Out of 26 bright lines measured, all have been identified with lines given in Professor Dyson's list (*Mem. R.A.S.*, vol. lvii., App., p. [1]). There are curious inconsistencies in the intensities of the bright lines on the plate; thus several lines of intensity 4 in Professor Dyson's list are not obviously present on the photograph, whilst other lines of intensities 2 and 1 are seen; but such inconsistencies are quite compatible with the recognised differences in the extension of emission lines beyond the limb.

No. 8 is a very complicated spectrum of emission lines and absorption lines, and shows features of great interest. In particular it is noticed that in several cases the bright lines are accompanied by dark companions, not represented in the ordinary Fraunhofer spectrum. The dark companions are on the violet side of the emission lines in so many cases that the temptation is to generalise and attach importance to the observation as possibly throwing light on the systematic displacements of lines attributed to different chemical elements; but the need for caution is well shown by the case of the two strontium lines at 4078 and 4216, in which it appears that the absorption companions are on opposite

sides of the emission lines. In the case of the scandium line at 4246·996 the displacement of the absorption line is approximately 0Å·44 to the violet.

The lines attributed in Rowland's Tables to zirconium attract attention in No. 8, inasmuch as, though they are in many cases not represented by emission lines, they produce a modification which may be described as the obliteration of the Fraunhofer absorption line.

The H_γ line appears in both No. 7 and No. 8 as a bright line with a dark central reversal. The H_δ line appears as a bright line in both cases without reversal. The widths are given as follows :—

	H_δ.	H_γ.	
	Width of Emission Line.	Width of Emission Line.	Width of Central Reversal.
	Å	Å	Å
In No. 7	0·686	0·878	0·353
In No. 8	0·946	0·934	0·313

On April 18, under similar conditions of cloudless sky, the spectrum of the Sun's limb was again repeatedly photographed with the same instrumental conditions as during the partial eclipse; but no actual bright lines were recorded. The hydrogen absorption lines, H_δ and H_γ, however, at once attracted attention by the great reduction in their width.

It is clear that exceedingly valuable work could be carried out with an instrument of high power by an observer who, in a total eclipse, stationed himself to the north or south of the band of totality at such a distance that the maximum phase was, let us say, 0·99. At such a station detailed observations could be conveniently made with a much more leisurely programme and with far greater completeness than on the central line.

Spectroscopic Observations during the Partial Eclipse of the Sun, 1912 April 17. By A. Fowler, F.R.S.

The partial eclipse of the sun, magnitude 0·92, was observed under very favourable conditions at the Royal College of Science, South Kensington. The 6-inch equatorial was employed, in conjunction with the Evershed spectroscope, for observations of the contacts by the spectroscopic method,* and for observations of the chromosphere and prominences. Times were recorded by the sidereal clock, the error of which was carefully determined by Messrs. Curtis and Jevons from transits of stars observed on April 16 and 17.

Mr. J. Storey, of the Royal Observatory, Edinburgh, was present during the eclipse, and confirmed all the observations

* The observations of the contacts are printed in Dr. Crommelin's summary of the results of various observers (p. 543).

except those of the contacts, which could only be made by one observer.

The Prominences.

Observations of the prominences were made before the eclipse commenced. There was a fairly bright prominence in the south-east quadrant, about 40″ in height, at position angle 116°; and a rather fainter pair of filamentary prominences in the south-west, nearly 60″ high, one with base at 192°, and the other at 201°. The corresponding heliographic latitudes were −52° E., and −52° W., −43° W. These prominences were observed in the same positions on the day before the eclipse, when their appearances were not very different. They were obviously of the quiescent type, and no bright lines, other than those of hydrogen and helium, were observed, except near the bases, where the *b* group and 5316·8 (1474K) were noted.

The prominences were beautifully displayed during the eclipse, and it was interesting to watch the passage of the Moon across them. Those in the south-west were eclipsed soon after first contact, and did not reappear until nearly a quarter of an hour after greatest phase. Their fine filamentary structure was then very well seen, as were also the bright lines in the spectrum at the base; the number of visible bright lines, however, was but little greater than in the observations before and after the eclipse.

The prominence −52° E. was eclipsed a little before maximum phase (12.10), and was observed to be just uncovered again at 12.57. It was somewhat brighter than the other prominences, and a slightly greater number of the usual "high level" chromospheric lines (5197·7, 5276·2, etc.) was seen. The metallic lines, however, were only observed close to the base, and such quiescent prominences are clearly of very simple chemical constitution.

The Chromosphere.

During the partial phases of an eclipse, when the angle at the cusp has become less than 90°, the chromosphere may be observed to greater advantage than at ordinary times, because its projection beyond the cusps is greater than the radial depth. The obliteration of low level chromospheric lines by superposition of the photospheric spectrum, produced by "boiling," may also be expected to be less effective at the cusps than in the usual observations at the limb. Some further advantage may be gained from reduced sky illumination. The length of chromospheric arc projecting beyond the cusps is of course greatest for the deeper strata,* and increases towards mid-eclipse on account of the smaller inclination of the limbs of the Sun and Moon at the points of intersection. In the undisturbed chromosphere, the bright lines

* The term "strata" is used for convenience of description.

would thus become visible in order of diminishing depth of the strata producing them.

It may be said at once that the visibility of the bright chromospheric lines greatly exceeded expectation. As early as 11.35, or about thirty-five minutes before maximum obscuration, the chromosphere was very brilliant at the southern cusp, and several of the high level lines were noted. Among the lines recognised at this stage were *b*, D, and the enhanced lines 5197·7 (Fe), 5226·7 (Ti), 5234·8 (Fe), 5276·2 (Fe), and 5316·8 (Fe), which, with the instruments employed, are not ordinarily observed unless there is at least a slight disturbance of the chromosphere. From 11.35 onwards, bright lines became visible in increasing numbers, exactly as in passing through metallic prominences of increasing activity. By 11.55, or fifteen minutes before mid-eclipse, the spectrum was very rich in bright lines at both cusps, and remained so for about half an hour. The lines were so numerous during this interval that it was impossible to record all of them, and the difficulty was increased by the dimness of the sky spectrum by which the identifications had to be made. Hundreds of the Fraunhofer lines must have been reversed bright, and the appearance of the spectrum strongly recalled that of the flash spectrum photographed during total eclipses.

Most of the bright lines were extremely short, but the high level lines were easily distinguished by their greater lengths when the slit was suitably placed. In some of the observations, the short lines at the cusps were curiously inclined to the dark lines of the sky spectrum. This is probably to be explained by the extreme thinness of the stratum in which the lines originate. If the image of the stratum were narrower than the slit of the spectroscope, the spectral lines would be images of the part of the stratum intercepted by the slit, and would therefore be inclined when the slit crossed the image obliquely. The thinness of the stratum would also account for the unusually high resolution in the bright line spectrum, which was strikingly evident.

About seventy bright lines between *b* and D were actually identified. It would serve no useful purpose, however, to give the complete list, as nearly all of them were included in the table of lines observed by me in a metallic prominence on 1905 February 11,* or in subsequent observations of chromospheric disturbances.† The majority of the lines also occur in Young's catalogue of chromospheric lines. The additional lines identified during the eclipse were reversals of well-marked Fraunhofer lines, due chiefly to iron and calcium. The barium line 5853·9 and other lines in the neighbourhood of D were also well seen.

From fifteen minutes after maximum phase the bright lines rapidly diminished in number, and the high level lines 5197·7, etc., were last observed at the cusps at thirty-five minutes after mid-eclipse, when the "magnitude" was reduced to about 0·55.

* *Monthly Notices*, vol. lxv. p. 517.
† *Ibid.*, vol. lxvi. p. 364.

Comparisons with photographs taken during total eclipses indicate that the chromospheric strata were successively observed when their projections beyond the cusps had reached about 13′ as measured along the Moon's limb. Thus, a stratum 3″ in depth would project by this amount at thirty-five minutes before and after mid-eclipse, and the enhanced lines 5197·7, etc., were first and last observed at those times. Similarly, the upper part of the flash stratum, if taken to be 1″·5 in depth, would project 12′ or more for about fifteen minutes on each side of mid-eclipse, and during this time the flash spectrum was finely displayed.

An arc of 13′ on the limb is equivalent to a radial depth of about 3″·5, but in ordinary observations, with the same instruments, lines originating at so low a level cannot be seen in the undisturbed chromosphere. That is, the chromospheric vapours were observed closer to the cusps during the eclipse than to the limb in everyday observations. Some benefit may have been gained from reduced sky illumination, but it seems probable that the chief advantage, in this respect, resulted from the smaller effect of unsteadiness at the cusps as compared with that at the limb.

The observations suggest that useful work on the chromospheric spectrum might be done during the partial phases of eclipses by the use of powerful spectrographs. In the present case a multitude of bright lines was visible for half an hour, when the magnitude of the eclipse ranged from 0·8 to 0·9, and long exposures might have been given if suitable apparatus had been available. Interesting results might in fact have been obtained during seventy minutes.

It may be added that a careful search for the green corona line was made near mid-eclipse, but no trace of the line was observed.

Dark Lines at Sun's Limb.

Other observations were made on the photospheric spectrum just within the Sun's limb, as the eclipse seemed likely to give a limb spectrum more than ordinarily free from superposed spectrum due to light from the central parts of the disc. It was found that the size of the image was too small for satisfactory observations of this nature, but by placing the slit as near the limb as possible, about the time of greatest phase, the titanium line 5219·9 was observed to be decidedly strengthened, and the iron line 5197·7 weakened, in accordance with the work of Mt. Wilson. A larger image and a powerful spectrograph might have yielded valuable results in this direction.

A general impression left by the observations is that partial eclipses, and the partial phases of total and annular eclipses, might be utilised for spectroscopic observations to a much greater extent than has hitherto been the case.

Summary of Observations of the Solar Eclipse of 1912 *April* 16–17. By Rev. C. D. P. Davies; J. L. E. Dreyer; Professor A. Fowler; G. J. Newbegin; Dr. A. A. Rambaut; Rev. W. Sidgreaves, S.J., and Rev. A. L. Cortie, S.J.; J. H. Worthington.

(*Epitomised by A. C. D. Crommelin.*)

The observations summarised in this report were made by the following observers:—

Observer.	Place.	N. Latitude.	Longitude.	Adopted Initial.
Rev. C. D. P. Davies	Maisons Laffitte, Paris	48° 56′ 45″	8m 32·5s E.	C.D.
J. L. E. Dreyer	Armagh Observatory	54 21 13	26 35·4 W.	J.D.
Prof A. Fowler, F.R.S.	South Kensington	51 29 50	0 42·4 W.	F.
G. J. Newbegin	Sutton, Surrey			N.
A. A. Rambaut, F.R.S.	Radcliffe Observatory, Oxford	51 45 35	5 2·6 W.	A.R.
W. H. Robinson				W.R
H. G. S. Barrett				B.
Rev. W. Sidgreaves	Stonyhurst Observatory	53 50 40	9 52·7 W.	S.
Rev. A. L. Cortie				C.
J. H. Worthington	4 km. N. of Ovar, Portugal	40 55±	34 20± W.	W.

C.D. was assisted by his son and by Mr. and Mrs. King Hall.

Instruments used:—

C.D. Small visual telescope, and the Waters telephoto lens (lent by the Council), mounted together as an equatorial coudé with a plane 12-inch mirror.

J.D. 10-inch refractor.

F. 6-inch equatorial in conjunction with Evershed spectroscope.

N. 9-inch Cooke equatorial. Power 75. Projection method employed.

A.R. 24-inch equatorial, stopped to 4 inches, used photographically.

S. and C. Browning spectroscope attached to 15-inch equatorial. The 4-inch Dallmeyer "Abney" lens, focal length 33 inches, was used to obtain photographs of the partial phase.

W.R. Barclay 10-inch equatorial. Power 85.

B. Marlborough $3\frac{1}{4}$-inch equatorial. Power 54.

W. Photographed flash spectrum with quartz prismatic camera. What instrument was used for visual observation of the corona is not stated.

Weather Conditions.

C.D. Fine throughout, with slight haze. Definition above the average.

J.D. Cloudy (except for one glimpse) till after greatest phase; then fine.

F. Very favourable throughout.

N. Fine; definition steady, except about last contact.

A.R. Very favourable. Overcast before eclipse, but almost cloudless during it.

S. and C. Fine with thin fog.

W. Fine. Very clear; calm.

Times of Contact, all expressed in G.M.T.

	First Contact.	Last Contact.	Remarks.
	d h m s	d h m s	
C.D.	April 16 22 49 8	17 1 32 23	Watch compared with Paris time signal.
J.D.		1 22 25·6	
F.	22 50 12	1 30 23	Spectroscopically observed on chromosphere. Considered accurate to 1 sec.
N.	22 50	1 29 35±	
W.R.	22 49 39·4	1 28 56·8	Both considered good.
B.	22 49 40·8	1 28 54·6	First contact perhaps 1 sec. late.
S.		1 27 59	Spectroscopically observed. Base of chromosphere re-appeared at point of contact.
C. (or assistant)		1 28 0·6	Observed on ground glass of camera. Set down 27 mins.

The spectroscopic method (first used by Young in 1869) is especially valuable at first contact, since an observation by the ordinary method must be late, there being no encroachment on the Sun at the time of geometrical contract.

F.'s observations on the chromosphere were made on the H_α line with tangential slit. He notes, "There was no difficulty in observing the moment of obliteration of the middle part of the visible chromospheric arc, which indicated first contact. For last contact the slit was placed across the cusps and kept there by the fine adjustments till the two bright H_α lines united."

W.R. and B. also noted the time of passage of a prominent lunar peak off the Sun's disc. Their times were $0^h\ 40^m\ 6^{\cdot s}0$ and $0^h\ 40^m\ 10^{\cdot s}7$ respectively.

Photographs.

Dr. Rambaut took 20 successful photographs in focal plane of equatorial. Sun's image 2·55 inches in diameter. Definition both of Sun's and Moon's limbs very good. Lunar mountains clearly seen. Last photograph only 7 seconds before last contact. Moon just discernible on it. The photographs are suitable for accurate measurement.

Plates were taken at Stonyhurst at the following times, the measured diameter of the Sun and distance between cusps being given.

G.M.T.	Diam. Sun.	Dist. between Cusps.	G.M.T.	Diam. Sun.	Dist. between Cusps.
h m s			h m s		
11 20 31·9	8·01	6·11	1 2 2·2	7·91	5·65
12 43 33·3	7·97	7·06	1 9 18·0	7·93	5·03
12 44 30·1	8·00	6·97	1 16 18·8	7·91	4·10
12 50 43·1	7·90	6·45	1 17 11·7	7·90	3·99

The first time is uncertain by 20 secs.; the others are exact.

Mr. Davies took one plate at mid-eclipse, and orientation exposures before and after. That at mid-eclipse, developed by Mr. P. Melotte, unfortunately shows a multiple image, probably due to the slow motion handle being touched during exposure. But Baily's Beads are well shown, isolated on the E. limb, beginning to run together on the W. They were also seen visually all round the disc. Chromosphere and corona not seen.

The Corona.

Mr. Worthington alone reports on this. He photographed the flash spectrum and observed the corona visually. It was seen faintly on W. side 3 sec. before totality; soon the equatorial streamers were seen as two brushes. The eclipse was total except for one bead of sunlight at N. Pole. For about 1½ second 4 coronal streams were seen, 2 to E. and 2 to W. That to N.W. was longest and strongest, extending 1 diam. from limb. The corona's general shape was that of a wind vane. The W. side disappeared first. Some part of the corona was visible for 10 seconds. He compares it to "a flower opening and closing." Pronounced minimum type, like that seen at Vavau in 1911. A sketch made just after totality is placed in the Library, also the photograph of the flash spectrum.

Prominences.

Father Cortie gives the following positions on April 16 and 17:

April 16d 22½h.			April 17d 1½h.		
No.	Position Angle.	Notes.	No.	Position Angle.	Notes.
	° ′			° ′	
1	39 0	Large.	1	94 20	Large.
2	46 40	Small.	2	115 15	Very large.
3	86 30	,,	3	117 40	
4	104 40	,,	4	190 0	Large.
5	113 20	Large, fine,	5	198 30	,,
6	115 20	prom.	6	203 40	
7	168 0	Small.	7	205 20	Small.
8	169 40		8	229 30	,,
9	190 40	Large, double.	9	321 0	,,
10	204 20	Medium.			
11	209 30	,,			
12	217 40	Small.			

These are likely to be of use for orienting photographs of the chromosphere or corona.

The disc was absolutely devoid of spots or faculæ at the time of the eclipse.

The Gloom.

C.D. says of mid-eclipse "It was as if a shadow had momentarily passed overhead." His party saw Venus, but not Mercury, which was, however, seen by Mr. Payn a few miles away.

R. says, "A duskiness or haziness prevailed like the symptoms of an approaching storm; in the Observatory the effect resembled early twilight. A hand lamp was required for the noon barometer reading, which can be sometimes made by daylight at 9 p.m. in June."

N. says a striking feature of the maximum phase was the very sombre appearance of the sky northwards.

S. and C. say during the maximum phase the aspect was just as if a heavy thunderstorm was imminent.

Fall of Temperature.

The readings communicated by C.D. were made by Mr. A. C. Allen.

	Previous Max.	Time.	Min.	Time.	Int. after Mid-eclipse.	Temp. at End of Eclipse.
	°	h m	°	m	m	°
C.D.	57½	23 15	53½	0 30	20	60
N (shade)	55	23 16	49	0 15	5	56
N (Sun)	67	22 51	54	0 10	...	63
S and C	48·5	22 0	46·2	0 0	...	47·6
W	62	22 39	48·4	23 48	5	60

N. says at mid-eclipse the easterly wind increased.

S. and C. note that the eclipse produced no appreciable effect on the magnetic elements.

1912 *May* 17.

The Spectrum of R Cygni. By the Rev. T. E. Espin, M.A.

The star was examined on several occasions in August and September 1911. When first seen it was about 13·5 mag. It was observed with various powers and seemed quite sharp; no nebulosity was seen, and the faint spectrum was apparently continuous. During September it was obviously increasing in light, and the spectrum appeared to be deeply banded, and was thought to be probably of Type IV. The weather was very cloudy in October, and no observation was obtained till October 27. From this date the results may be quoted in detail:—

Oct. 27, mag. 8·7, Type III.!! The bands well seen.

Oct. 28. The bands quite clear, large and deep, and Type is III.!!

Nov. 2, mag. 7·6. The star has brightened greatly; the bands do not seem so pronounced, and the star does not look so red. F was looked for, but not seen.

Nov. 6, mag. 7·5. The bands seem less distinct, and F is bright but not easy to see.

Nov. 10, mag. 7·3. The star is certainly III. Dunér's bands 2, 3, 4 are very strong; after that the bands are much fainter. F is now clearly seen.

Nov. 16, mag. 7·0. There is little alteration. The most prominent object is F. The bands are generally feeble, and ill seen, and the spectrum III. only and not more. There is a bright line in the yellow, and the bands at this end of the spectrum are the most pronounced. C is also bright.

Nov. 29, mag. 6·8. The type is III., but not conspicuously so. F is the most prominent feature in the spectrum, but is not so bright.

Dec. 3, mag. 6·8. Twilight and Moon and passing clouds. There are bands, and the spectrum may be III., but uncertain. F only glimpsed.

Dec. 9, mag. 6·8. Definition not good. The spectrum long and carefully observed. F is still bright, but difficult to see. Bands in the red are strong. Dunér's Nos. 1 and 2 well seen. A bright line near D? Nos. 3, 4, 5, 6, 7, 8 are faint; 9 and 10 seem one large band. The spectrum appears to be III., but ill-defined.

Dec. 14, mag. 7·1. The star is distinctly fainter. The bands in the yellow and red are quite obvious, and the spectrum appears to be III. F is only glimpsed at times, and there seems a bright line near D. The violet end is strongly absorbed, and the spectrum in the blue and violet looks like IV. The star is redder.

The analysis of the above observations shows: that

(1) The bands became less obvious as the star brightened.

(2) The visual maximum derived from the brightness of F took place on November 21.

(3) The actual maximum took place on December 5.

It may be well to collect here all the spectroscopic observations of this object.

It was observed by Dunér, 1879 March 16, 1880 May 25, 1882 October 6, and was noted as III.! "The bands in the blue are very large and rather deep as far as band 9 inclusive." On 1888 August 13, I observed F as a very bright line, and the Third Type spectrum was not certainly seen. Dr. Copeland measured F, and believed the spectrum to be III. type. Mr. Maunder (*M.N.* xlix. p. 306), on September 21, found bright lines near D_3 and F, and on October 1 only F. He adds that the general spectrum of the star, apart from the bright line at F, appeared to be Secchi's Fourth Type. On 1889 September 16 I found the star mag. 7·5 and Type III., and F was not certainly seen. On October 28 Dr. Becker found F very bright. Mrs. Fleming, 1890 November 19, found Hδ Hγ bright on the Harvard photographs; but 1904 December 7 the star was Type IV., and there was no trace of the hydrogen lines. It is stated in *Harvard Annals*, lvi. No. 6, that "the spectrum is peculiar and shows changes in the various photographs," p. 211; and on p. 224, "On the best photograph this spectrum resembles class N."

It may be well to add that Mr. Grover has at times seen a small nebula in place of the star at minimum. During the earlier observations in 1911 the star was carefully examined with the 17¼-inch and showed no sign of nebulosity as before stated. The spectroscope never showed a nebular spectrum. But Mr. Grover informs me that the nebula was seen by him in 1895, 1897, 1898, 1900, and adds, "During the last fourteen years this nebula has been many times observed when the star was *invisible*. I have always noticed that as soon as the faintest sign of the star showed itself the nebula at once vanished." There is so much that is obscure about the spectrum of this star that I asked Professor Wright of the Lick Observatory to photograph it, and he has very kindly done so. Unfortunately the maximum occurred earlier than we expected, and so the photographs were taken after it. The account of these plates and his measures are communicated (p. 548). One or two points deserve particular attention.

(1) Dunér's bands for Types III. and IV. were drawn to the scale of the enlarged photographs and compared directly with the plate. There is no apparent agreement with Type IV. With Type III. there is better agreement, although, as might be expected, the photographs show much additional detail. Band 1 begins at wave-length shorter than in Dunér's measures, and is not so extended. Band 2 is more extended in both directions; a very dark absorption band between 1 and 2 seems to coincide with Dunér's dark ray in α Orionis λ6337. Band 3 is well defined. Band 4 is of vastly

greater extent and more complicated. If, however, Keeler's band in α Orionis $\lambda 5669$ to $\lambda 5682$ is added, and the photograph of μ Geminorum (*Publication of the Yerkes Observatory*, vol. ii. No. 5, plate 10) is studied, it will be seen that the result is very similar in R Cygni, save that the commencement is slightly more refrangible. From this point to Hβ the spectrum becomes such a mixture of bright and dark bands that it is no longer possible to distinguish the Third Type spectrum with certainty.

(2) Of the bright lines other than hydrogen, the one at $\lambda 6289$ is the most remarkable. The helium line $\lambda 5016$ is apparently bright with a dark line on the more refrangible side. This is probably the case with $\lambda 4922$.

(3) In the second enlargement, containing the region between Hβ and Hγ, next to the brilliancy of these lines the most remarkable object is the dark band whose centre is $\lambda 4644$. This has a striking similarity to the one in γ Cassiopeiæ as given by Sidgreaves (*M.N.*, lix. No 8, plate 9). Indeed, the spectrum in this region appears to have a certain resemblance in other details to that of γ Cassiopeiæ.

(4) The variation in the more refrangible hydrogen lines, mentioned by Professor Wright, finds its counterpart in the behaviour of Hα. It seems probable that when the maximum approaches the lines slowly brighten up in order of refrangibility, till at length the climax is reached with the brightening of Hα. By this time the more refrangible lines have faded down; then with the fading of Hα the lines of greater refrangibility brighten up again in order.

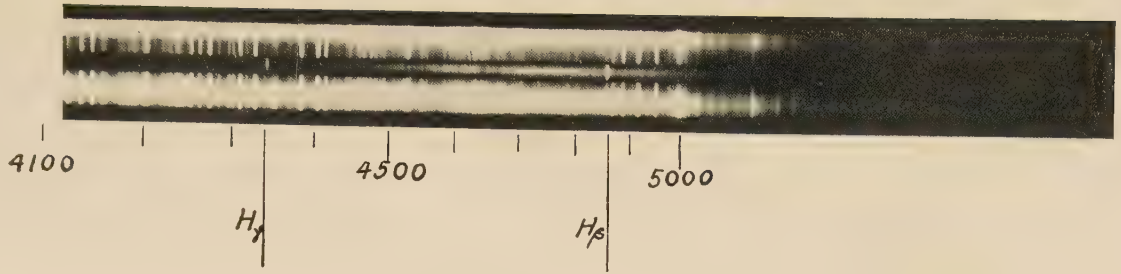
The Spectrum of R Cygni. By W. H. Wright, Astronomer at the Lick Observatory. (Plate 8.)

(*Communicated by the Rev. T. E. Espin.*)

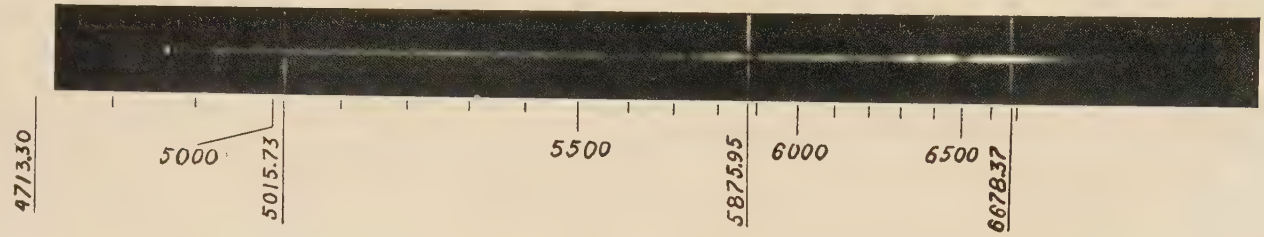
With the hope that they will not be too late to accompany your paper on the spectrum of R Cygni, I am enclosing my measurements of the three spectrograms it was found possible to secure. These plates were photographed with a single-prism spectrograph attached to the 36-inch refractor. I greatly regret the inability to get a more extensive series of photographs, but we have had bad luck with our weather this winter. There has been practically no observing.

You ask for my opinion of these plates: it is hard to form an opinion of a variable star that is worth much, on the basis of three observations; but, speaking generally, a very remarkable feature appears to be that the continuous spectrum fades out in the blue. With most stars the photographic maximum is in the neighbourhood of 4500 A, but that is the point where the continuous spectrum of R Cygni ended, at the time of these observations.

Connected with this is the comparative faintness or absence of



SPECTRUM OF R CYGNI. 1911 DEC. 5.



SPECTRUM OF R CYGNI. 1911 DEC. 11.—W. H. WRIGHT, LICK OBSERVATORY.

the violet and ultra-violet members of the hydrogen series of bright lines. On the plate of December 5, H_γ is very bright, but there is only a trace of H_δ. On the plate taken 1912 January 13, H_γ is possibly a trifle fainter than on the former occasion, but H_δ is quite strong. No lines have been observed on the more refrangible side of H_δ.

These facts are strongly suggestive of absorption by an extensive atmosphere surrounding the star, and this absorption may well play an important part in causing the fluctuations of its light. It is easy to explain the absence of violet and ultra-violet light in the continuous spectrum by assuming a comparatively cold star, but here is an apparently perfectly good hydrogen series cut off in its prime above H_γ on December 5, and above H_δ on January 13. Under normal conditions the lines in such a series fade away gradually in intensity, and it therefore looks as though the seat of bright line radiation were surrounded by a medium exerting a powerful general absorption in the violet and ultra-violet. Furthermore, the increase in intensity of H_δ would seem to show that the amount of absorption is variable. Similar considerations may well apply to the continuous spectrum.

The notion that we are dealing with extensive *general* absorption in this star receives some support from the fact of the existence of great selective absorption as shown by the great breadth of most of the dark lines. For instance, the D lines form a blend extending from 5883 to 5902A, and others are perhaps equally broad.

One is tempted to speculate on the possible causes of such variations in absorption as are suspected, but three observations should not be made the basis of a variable star theory, and I shall have to rest content until it is possible to secure more. You have certainly interested me in this star, and I hope to observe it in the future.

The spectrum, as you see from the plates sent you, is considerably confused. It is impossible to measure everything in it, or to know in every case just what you are measuring. If you see anything that I have not measured you can get its approximate position, from your positives, with the aid of the paper scales which accompany this letter.

H_α and H_β (C and F) appear to be accompanied on their more refrangible sides by dark lines. Whether H_γ and H_δ are similarly provided with companions cannot be determined, as the continuous spectrum is too faint to photograph.

All of the hydrogen lines appear to be displaced negatively, as shown by the following comparison:—

Vacuum Tube.	R Cygni.	*–Tube.	Velocity.
4101·8	4101·4	– ·4A	– 30 km.
4340·6	4340·2	– ·4	– 28
4861·5	4860·7	– ·8	– 50
6563·0	6561·3	– 1·7±	(– 78±±)

The dispersion for the last line is so small that the residual is hardly significant. The others harmonise very well with the assumption of a velocity of approach of 30 or 40 km. per second.

It is to be regretted that it was impossible to photograph the H_β line under high dispersion. This would have been an easy matter, but the weather did not permit.

I have not had time to check my computations on which the wave-lengths given in the accompanying tables depend, but believe them to be free from serious errors. The small corrections to allow for the orbital motion of the Earth have been applied.

R Cygni.—Measurements of Spectrum.

1911 December 5. Plate No. 345.		1912 January 13. Plate No. 349.	
4101·7±	Suspect trace of bright line.	4101·4	Pretty strong bright line. H_δ 4101·85.
4340·2	Strong bright line.	4340·2	Strong bright line. H_γ 4340·66.
4447	Suspect very faint maximum.		
4497	Dark line in very faint continuous spectrum.		
4521·1	Narrow maximum.		
4530·1	Narrow maximum.		
4538·3	Faint narrow maximum.		
4549·9	Brightish maximum on border of dark line. Spectrum very faint above this point.		
4553·6	Dark line. Intensity = 3.		
4563·0	Hazy dark line. $i=2$.		
4571·7	Dark line. $i=1$.		
4579·7	Dark line. $i=2$.		
4583·3	Narrow maximum, possibly bright line.	4583·7	Narrow maximum, probably a bright line.
4592	Hazy dark line. $i=1$.		
4606·2	Dark line. $i=2$.		
4620·9	Dark line. $i=1$.		
4631·6	Narrow maximum (bright line ?)	4630·6	Narrow maximum, probably a bright line.
4633·7	Narrow dark line. $i=1$.		
		4638	Suspect maximum.
4640 } 4644·4 } 4648	Edge. Broad dark line. $i=5$. Edge.		
4651·2	Narrow maximum (bright line ?).	4651·1	Narrow maximum, probably bright line.
4655	Dark line. $i=1$.		

R Cygni.—Measurements of Spectrum—continued.

1911 December 5.
Plate No. 345.

58·3	Dark line.	$i=2$.
63·5, 90·7	Double dark line.	
40·8	Dark line.	$i=1$.
53·2	Dark line.	$i=1$.
60·8	Very bright line.	
93	Dark line.	$i=1$.
20	Dark line.	$i=1$.
56	Dark line.	$i=1$.
89·8±	Dark line.	$i=3$.
06	Suspect dark line.	

1912 January 13.
Plate No. 349.

4795	Maximum of suspected hazy bright band.
4839·7	Dark line.
4853·9	Dark line.
4860·8	Very bright line. Hβ 4861·50.
4928	Hazy maximum.
4958	Suspect dark line.
5012	Dark line.
5038·3±	Dark line.

1911 December 11.
Plate No. 346.

4860·6	Strong bright line (Hβ)
4913	Dark line.
4921	Dark line.
4925	Dark line.
4932	Dark line.
4957	Dark line.
5014	Dark line.
5017	Slight maximum.
5022	Hazy dark line.
5039	Fairly sharp dark line.
5065	Suspect narrow dark line.
5149	Broad dark line.
5207	Dark line.
5224	Dark line.
5239	Dark line.
5249	Dark line.
5267	Dark line (looks double).
5283	Suspect dark line.
5297	Dark line.
5310	Dark line.
5356	Bright line or band?

R Cygni.—Measurements of Spectrum—continued.

1911 December 11—*continued.*
Plate No. 346.

5360±	Between these limits there is considerable detail in the way of dark lines, which it is impossible to measure satisfactorily.
5500±	
5675±	Between these limits spectrum is very faint.
5713	Maximum, or bright band.
5767	Broad bright band. Maximum on less refrangible side of centre.
5851	
5883	Edge.
5893·0	*Very* strong, broad, dark line. $D_2 - D_1$ 5890·19–96·16.
5902	Edge.
5955	Suspect fairly narrow dark line.
6059	Hazy dark line.
6138	Broad dark line.
6160	Broad dark line.
6209	Hazy dark line.
6237	Hazy dark line (looks double).

Below this point the spectrum is rather hard to describe in a table. The wave-lengths have therefore been attached to a sketch.

The indicated breadth of H_a does not mean that it is not monochromatic. The bright hydrogen lines, where not over-exposed, are about as broad as the comparison lines. 6289 looks like a monochromatic line.

The solid line sketch was made with the plate under the microscope. Subsequently, on examination of the negative with a lower power, details seem to be brought out which have been inserted in broken lines. The sketch is based on measurements of the negative, and the dispersion is therefore prismatic, so you can compare it directly with your positives—allowing, of course, for difference in scale.

Preliminary Observations of Spiral Nebulæ in Polarised Light.
By J. H. Reynolds.

A possible explanation of the stellar spectrum given by the great majority of spiral nebulæ is that some proportion of the light may be reflected from a central star or stars, involved in the nucleus. Long-exposure photographs tend to blot out the true character of the nucleus, which is usually globular in form with the maximum of light at the centre. The main reasons which favour the adoption of this as a provisional hypothesis may be enumerated as follows:—

1. The few cases in which spectrograms of spiral nebulæ have been secured include one at least where the continuous spectrum is crossed by bright lines of nebulium and hydrogen (N.G.C. 1068, 5-foot reflector, Mount Wilson). Sir William Huggins also recorded the existence of bright lines seen visually in the spectrum of the Andromeda Nebula (*Atlas of Representative Stellar Spectra*). The analogous character of cometary spectra is apparent.

2. In some authenticated instances of variable nebulæ, the nebulæ have been situated in close proximity to variable stars (T Tauri, R Coronæ Australis, Nova Persei).

3. Where spiral nebulæ are seen nearly edgewise, as is evidently the case with such examples as N.G.C. 3628, they are crossed by a dark band. It may be inferred that the matter forming the outer portions of the spiral is not sufficiently illuminated to affect the photographic plate, but is dense enough to absorb the light having its origin in the stellar nucleus.

It was suggested by Mr. F. J. M. Stratton that the hypothesis might be tested by means of polarised light, and Prof. Newall further proposed that a double-image prism should be used for photography. Both these valuable suggestions were adopted.

The first attempt to apply this test was made with a Nicol prism and positive eyepiece loaned by the Society, used in conjunction with the 28-inch reflector at Harborne. Some indications of

polarisation were made out with this apparatus; but the observations were complicated by the polarisation introduced by the 45° plane of the reflector, so that it was impossible to be certain whether the polarisation lay in the nebulæ or the mirrors.

A double-image prism was therefore substituted for the Nicol. This has the great advantage that the two images of the nebula can be seen side by side in opposite polarisations and the polarisation introduced by the telescope itself is nullified.

The nebula which was chosen as most suitable for this purpose was M64. This object, which is in Coma Berenices, is one of the brightest and largest of this region, and is inclined at a considerable angle to the line of sight: there is also a star of about the same magnitude as the nucleus, which lies a little to the north of the nebula.

The double-image prism seemed to show a decided difference in the visual appearance of the two images as the prism was rotated, although they were, of course, very faint. To put the matter to a final test, however, it was decided to employ photography. The apparatus was arranged so that the prism could be placed in the focal cone, the two images being brought to a focus about 5 millimetres apart; this allowed the full aperture of the reflector to be used. The prism could also be rotated to bring the axis through either the major or minor axes of the two nebular images.

Several nights were spent in experimenting as to the position of the best focus, and in ascertaining the amount of distortion introduced by the prism.

It was found that the best focus of the polarised images lay about 2 millimetres behind the normal plane, and that the two star images were distorted in different directions. It also became evident that no exposure of less than $2\frac{1}{2}$ hours would under ordinary circumstances bring out sufficient density for the images to be examined.

A photograph with this length of exposure was obtained on 1912 April 11, with the axis of the prism passing through the minor axes of the nebular images. On examination there was seen what appeared to be a real difference between the two nebular images. One of the images (N1) exhibits its maximum brightness at right angles to the axis of the prism: the other (N2) is nearly circular in its brightest part, but has been considerably distorted in the direction indicated by the corresponding star image: if allowance is made for this distortion, the second image would appear to have its maximum brightness in a direction at right angles to the first. This effect would be expected if polarisation were present under the conditions required by the hypothesis. The image N1 is appreciably fainter as a whole than N2, but as this difference extends to the corresponding star images it is undoubtedly instrumental in its origin. The question of the reality or otherwise of the polarisation effect would be much simplified if the apparatus were employed on a larger and brighter spiral, where the amount of distortion of the images would be

small compared with their size: in such a case the polarisation would be plainly revealed if present. It is proposed therefore to make a further test of the hypothesis when the Andromeda Nebula is accessible.

I should like to record my obligations to Mr. Stratton and Prof. Newall for their assistance and advice.

Note added 1912 *May* 23.

But, in addition to the polarisation effect, there is another effect we should expect to find if the hypothesis is correct. If we take the case of a spiral nebula the plane of which lies at an angle of 45° or less to the line of sight, the reflection from the earthward half of the nebula would be oblique, with the maximum of obliquity along the minor axis of the ellipse. On the other hand, the reflection from the further half would be acute, so we should look for the earthward half to be the brighter of the two. Over 45° this brightening would not be marked, but the effect would increase as the angle with the line of sight decreased. An examination of photographs of spiral nebulæ confirms the existence of this feature: among those showing it conspicuously are N.G.C. 224, 2683, 2841, 3031, 4501, and 7331: in the case of N.G.C. 2683, where the inclination is only about 10°, one-half of the ellipse is almost invisible. There is a point in this connection, however, which seems rather significant: the oblique illumination effect is apparently almost non-existent in nebulæ of the hard, granular type, all the examples mentioned above being mainly of the soft, filmy type. There is evidently a fundamental distinction between these two types, although both are sometimes found in the same nebula.

On the Motions and Distances of certain Stars of the Types B8 and B9. By H. C. Plummer, M.A.

1. The apparent general success which was met with in applying the hypothesis that the stars of type A are moving parallel to the plane of the Milky Way to an investigation of their motions and distances gives an incentive to the attempt to extend the inquiry to other classes of stars. In general we cannot expect the same measure of success, since the radial velocities of other types have not revealed the property which formed the starting-point of the previous investigation—namely, that the larger components of the velocity in the line of sight are associated with the stars in the lower galactic latitudes. But there are certain stars—those of types B8 and B9—for which satisfactory results appeared quite possible, because Mr. B. Boss has suggested that these stars, in virtue of their proper motions, are to be connected rather with the A type than with the other B type stars. It seemed therefore worth while to make this supposition, and the results appear to justify this course. The material was taken from a MS. list of radial motions of the

B type stars, very kindly sent by Dr. Campbell at the same time as the data for the A type stars already discussed; but the figures differ in only two or three instances from those published in *L.O.B.*, No. 195.

2. An important point suggests itself at the outset. What value are we to assign to the constant error K, which was found by Dr. Campbell to affect the observed radial velocities, for the class of stars in question? No independent determination of K seems to have been made, the number of stars available being small. But we have the results (*L.O.B.*, No. 195, pp. 107, 116), $K = +3{\cdot}94$ km. for 225 stars of class B in general, and $K = +4{\cdot}70$ for the 180 stars left after excluding 45 stars of the types B8 and B9. Hence we can infer that the value sought is approximately

$$K = 5 \times 3{\cdot}94 - 4 \times 4{\cdot}70 = 19{\cdot}70 - 18{\cdot}80 = +0{\cdot}90 \text{ km.}$$

Now, this value of K happens to be very close to the value found for the A type stars, namely $+0{\cdot}95$ km. Thus the association of the B8 and B9 stars with type A rather than with type B is confirmed by the value of the systematic error which seems to be shown by their observed radial velocities. For the A type stars K was assumed to be +1 km., and we are justified in taking the same value for the stars now under discussion.

The method followed is exactly the same as that described in *Monthly Notices*, lxxii. p. 176, and more fully in a forthcoming *Lick Observatory Bulletin*. The Sun's motion is assumed to be 16·8 km./sec. in the direction of $A = 270°$, $D = +30°$, as determined for the A type stars in *L.O.B.*, 196. This gives for the components towards the First Point of Aries, the point on the equator 6^h in advance and the North Pole 0, $-14{\cdot}55$ and $+8{\cdot}4$ km. The component normal to the Milky Way, of which the pole is taken at $(192°, +27°)$, is therefore 6·50 km. towards this pole.

3. The results are given in Table I. The number in the first column is that which the star has in the *Preliminary General Catalogue* of Boss, from which the proper motions in the eighth and ninth columns are taken. The type and magnitude in the third and fourth columns are taken from the *Revised Harvard Photometry*. It should be noticed that β Monocerotis was there assigned to the type B3*p*, whereas among the Lick results it is transferred to B9; its inclusion here is apparently justified so far as the deduced motion is concerned. The approximate position of each star for 1900·0 is given in the sixth and seventh columns. The quantity V in the tenth column is the observed radial velocity in km./sec., and is taken from a MS. list, which, however, as already stated, differs in very few instances from the results published in *L.O.B.*, 195. The eleventh column gives under the heading ϖ the deduced parallaxes, and the last three columns the corresponding components of linear velocity in km./sec., u being directed towards the First Point of Aries, v towards the point $(6^h, 0°)$ and w towards the North Pole. These are relative to the Sun. The numbers

TABLE I.

No.	Stars.	Type.	m	m_0	R.A.	Decl.	μ_α	μ_δ	V	ϖ	u	v	w
					h m	° ′	s	″		″			
100	β^1 Tucanae	B9	4·5		6 45	−63 31	+·0131	−·054	0·0	neg.			
257	ϕ Androm.	B8	4·3	2·2	15 56	+46 42	+·0007	−·007	− 4·0	·0038	+ 0·4	+ 9 5	− 9 7
369	τ ,,	B8	4 9	5·4	23 40	+40 4	+·0012	−·025	− 2·0	·0126	+ 1·4	+ 6·3	− 9·1
433	ϕ Phœnicis	B9	5·0		27 33	−43 0	−·0038	−·032	+13·0	neg.			
580	ω Fornacis	B9	5·0		37 22	−28 40	−·0017	−·018	+10·9	,,			
621	ϵ Hydri	B9	4·3		39 32	−68 42	+·0167	+·016	+ 5·5	,,			
708	β Persei	B8	var.	−3·8	45 26	+40 34	+·0006	− 005	+ 4·1	·0007	−17·0	+ 53·0	−25·4
781	Pi 51	B9	4·4		50 15	+59 36	+·0005	−·001	− 5·0	neg.			
786	Pi 54	B9	4·8		50 28	+58 32	+·0014	·000	− 6·0	,,			
877	27 Tauri	B8p	3·8	7·8	55 48	+23 45	+·0014	−·050	+13·0	·0636	+ 5·8	+ 11·2	+ 1·4
885	f Eridani	B8	4·9		56 14	−37 55	+·0068	−·018	+18·0	neg.			
896	Pi 177	B9	4·9		57 9	+62 47	+·0008	+·002	+ 5·0	,,			
1001	41 Eridani	B9	3·6	6·4	63 32	−34 3	+·0047	−·003	+17·4	·0370	− 0·8	+ 15·3	− 9·5
1250	β Orionis	B8p	0·3	−4·0	77 26	− 8 19	+·0001	−·001	+22·6	·0014	− 0·6	+ 21·5	− 6·6
1304	β Tauri	B8	1·8	5·7	80 0	+28 31	+·0024	−·177	+11·0	·0600	+ 0·2	+ 15·6	− 7·5
1438	134 Tauri	B9	4·9	7·4	85 58	+12 37	−·0009	−·026	+21·1	·0313	+ 3·4	+ 20·3	+ 0·5
1566	υ Doradus	B9	5·2		92 20	−68 49	−·0109	+·032	+25·0	neg.			
1639	β Monoc.	B9	4·7	7·9	96 0	− 6 58	−·0025	·000	+20·6	·0433	+ 2·0	+ 19·8	− 2·4
1702	ν Puppis	B8	3·2		98 40	−43 6	+·0001	−·020	+26·0	neg.			
1924	Lal. 14382	B8	4·9	4·0	109 27	−18 50	−·0005	−·004	+17·0	·0065	+ 0·1	+ 15·1	− 7·9
2011	k Puppis	B8	4·5		113 40	−26 34	−·0008	+·024	+23·0	neg.			
2406	c Carinæ	B8	4·0		133 12	−60 16	−·0034	+·057	+24·5	,,			
2445	κ Cancri	B8	5·1	3·2	135 36	+11 4	−·0013	−·011	+26·3	·0041	− 4·0	+ 34·9	− 7·7

TABLE I.—*continued.*

No.	Stars.	Type.	m	m_0	R.A.	Decl.	μ_α	μ_δ	V	ϖ	u	v	w
					h m	° ′	s	″		″			
2604	*m* Carinæ	B9	4·7	4·8	144 9	−60 53	−·0047	+·019	+26·0	·0105	− 6·9	+ 24·1	−17·6
2790	δ Antliæ	B9	5·6	5·4	156 15	−30 5	−·0034	−·010	+25·0	·0090	− 7·2	+ 28·6	−16·6
3048	A Centauri	B8	4·8	5·4	172 30	−53 42	−·0073	−·003	+12·0	·0132	− 2·6	+ 23·8	− 9·5
3054	λ ,,	B9	3·3	5·5	172 48	−62 28	−·0060	−·022	+11·0	·0248	+ 0·1	+ 8·0	−10·8
3115	β Hydræ	B9	4·4		176 57	−33 21	−·0045	−·006	−20·0	neg.			
3191	γ Corvi	B8	2·8	7·5	182 40	−16 59	−·0112	+·011	− 7·0	·0886	+ 7·1	+ 8·9	+ 2·9
3300	*l* Centauri	B8	4·8	6·0	188 36	−39 26	−·0040	−·043	+13·0	·0176	− 3·7	+ 12·1	−16·6
3305	L 5241	B8	5·0	4·5	189 3	−59 8	−·0028	−·012	+19·5	·0079	− 5·3	+ 12·2	−19·6
4089	υ Herculis	B9	4·6	5·5	239 54	+46 19	+·0055	−·068	+ 4·9	·0150	+ 6·5	− 24·9	−12·1
4094	ι^2 Normæ	B9	5·8	7·0	240 15	−57 40	−·0038	−·058	+ 6·0	·0176	− 1·9	+ 13·2	−12·6
4498	L 7451	B8	4·8	4·8	265 40	−31 40	+·0002	−·024	−16·0	·0100	+ 2·8	+ 20·3	− 0·8
4637	L 7677	B8	5·4	5·5	274 2	−36 43	+·0009	−·021	−11·5	·0103	+ 3·9	+ 16·1	− 0·3
4666	ν Pavonis	B8	4·8		275 30	−62 20	−·0010	−·033	+62·0	neg.			
4796	λ Telescopii	B9	5·0	4·2	282 38	−53 4	+·0020	+·014	− 4·2	·0070	+13·0	− 1·7	+ 9·9
4929	β^1 Sagittarii	B8	4·2	0·8	288 51	−44 39	−·0001	−·019	− 4·0	·0021	−13·1	+ 31·0	−26·8
4934	υ ,,	B8*p*	4·6		289 0	−16 8	−·0002	−·004	−10·0	neg.			
4999	52 ,,	B9	4·7	6·8	292 39	−25 6	+·0053	−·025	−17·0	·0268	+ 4·7	+ 21·7	+ 3·6
5199	κ Cephei	B9	4·4	−0·4	303 4	+77 25	+·0038	+·026	−20·9	·0011	−17·7	+127·2	+ 3·5
5288	υ Pavonis	B9	5·4	4·1	308 12	−67 7	+·0016	−·016	+ 5·7	·0056	− 0·4	+ 13·3	− 9·6
5310	α Delphini	B8	3·9	5·5	308 45	+15 34	+·0044	−·008	−22·0	·0207	2·2	+ 26·0	− 7·9
5349	Pi 305	B8	5·8	1·5	310 51	−26 9	−·0002	−·030	−13·5	·0014	−44·6	+ 37·7	−84·6
6054	β Sculptoris	B9	4·5	6·6	351 54	−38 22	+·0077	+·011	+ 2·3	·0265	+ 4·5	+ 15·7	+ 0·7

under m_0 in the fifth column indicate absolute magnitude at a uniform distance corresponding to the parallax 0″·01, being calculated by the formula

$$m_0 = m + 5 \log (\varpi / 0''{\cdot}01).$$

4. In all, the results for 45 stars appear in Table I., and of these 30 stars give positive parallaxes and 15 stars, or exactly one-third, negative. The largest parallax, that of γ Corvi (mag. 2·8) is less than 0″·09, and it is rather the smallest parallaxes which seem open to suspicion. This applies in particular to Algol, where the proper motion is small and a small error would have a comparatively large effect. In three cases the parallax has been directly determined, with the following results:—

Table II.

Star.	ϖ (hyp.)	ϖ (meas.)	Reference.
	″	″	
β Persei	·0007	+·058	Astr. Obsns. Oxford, iv. (2)
		+·037	Trans. Astr. Obs. Yale, ii. (1)
		+·122	Flint, A.J., 631
β Orionis	·0014	+·055	Schur, A.N., 3033
		·000	Annals R. Obs., Cape of Good Hope, viii. (2)
β Tauri	·0600	+·063	Astr. Obsns. Oxford, iv. (2)
		−·073	Flint, A.J., 631

It is clear that this kind of comparison gives evidence of little value either for or against the hypothetical parallaxes, for it is both scanty and inconsistent.

5. The means of the components (u, v, w) determine the average drift of the stars whose parallaxes come out positive, relative to the Sun. Of the 30, 2 clearly stand out from the rest, κ Cephei (see § 8 below) and Pi 305. For the remaining 28 the means are *

$$u = -0{\cdot}34, \qquad v = +16{\cdot}17, \qquad w = -8{\cdot}38,$$

which, when reversed, give for the relative motion of the Sun

$$\mathrm{R} = 18{\cdot}2 \text{ km./sec. towards } \mathrm{A} = 271^\circ{\cdot}2, \quad \mathrm{D} = +27^\circ{\cdot}4.$$

in satisfactory agreement with other determinations. Subtracting the means from the individual components, we can determine the average internal components—namely:

$$\Delta u = \pm 4{\cdot}4, \qquad \Delta v = \pm 9{\cdot}1, \qquad \Delta w = \pm 6{\cdot}3.$$

There are, however, two stars which give very discordant results in the v component, and yet approximately balance one another in

* A correction has been made in the radial velocity of 134 Tauri after the means in § 5 had been calculated. The changes would not be material, and it has been thought unnecessary to repeat the calculation.

the determination of the means. These are β Persei and υ Herculis ($\Delta v = +36{\cdot}8$ and $-41{\cdot}1$ respectively), which disturb the average of the residuals (without sign) considerably. If they are removed, the average of the remaining 26 residuals, Δv, falls to $\pm 6{\cdot}8$, in better agreement with Δw. Thus the average residual motion in any random direction for the majority of these stars is, roughly speaking, of the order ± 6 km./sec., always understanding that the component in one direction, normal to the Galaxy, has been made zero. Now, this figure agrees with Campbell's residual radial velocities after correction for the solar motion, found from the B type stars grouped in a variety of ways (*L.O.B.*, 195, p. 119). Thus our present hypothesis has in no way exaggerated the internal motions of the majority of the stars of the class considered, as a false assumption would almost certainly do. We now see more clearly than before that this class of stars holds a middle position between the earlier B type stars and the A type stars not only in respect of spectral type but also of intrinsic motion. As regards proper motion and preference for motions parallel to the plane of the Milky Way, the stars of type B8 and B9 are more nearly allied to the A type stars; as regards the order of magnitude of their average velocities they preserve the properties of the B–B5 stars, which are more distant (twice as distant according to Campbell) and intrinsically brighter.

6. We may next consider the absolute magnitudes m_0 which the stars would have if placed at the distance corresponding to a parallax $0''{\cdot}01$. The means are:

B8	17 stars	mean $m_0 = 3{\cdot}6$
B9	13 ,,	,, $m_0 = 5{\cdot}4$

If we combine them and add for comparison results previously derived for the A type stars, we obtain:

TABLE III.

Type.	No. of Stars.	Mean m_0.
B8–B9	30	4·4
A	56	5·4
A1–A3	37	6·3
A5–A8	19	7·6

The progressive decline in brightness with advancing type is very clearly marked. If we assume that the actual magnitude of the Sun is $-26{\cdot}6$, its absolute magnitude on the above scale is, conveniently enough, 10.

It should be noted that the absolute magnitudes given above for the several sub-types are all probably too bright. This we must infer because, irrespective of distance, the exceptionally bright star has a better chance of being observed, while the exceptionally faint star has a poor chance of appearing in our list.

This effect is very noticeable if the absolute magnitudes are arranged in an ordered sequence.

7. The variability of the absolute magnitudes for the present class of stars is not very great, 60 per cent. ranging between 4 and 7. In consequence we might hope to find some correlation between the apparent magnitudes and the deduced parallaxes. And this seems to be the case. The mean values are as follows :—

TABLE IV.

Limits of m.	No. of Stars.	Mean ϖ.	ϖ'	$\varpi - \varpi'$.
2·5–3·5	3	″·0380	″·0362	+ ″·0018
3·5–4·5	7	·0221	·0229	− ·0008
4·5–5·5	15	·0148	·0144	+ ·0004
5·5–6·0	3	·0093	·0102	− ·0009

In the fourth column is given a parallax computed from the formula :

$$\log \varpi' = 8{\cdot}5593 - 0{\cdot}2(m - 3{\cdot}0),$$

taking m at the centre of the interval given in the first column. The differences in the fifth column seem to show that the brightness of these stars does depend very largely on their distances according to the normal law for stars of uniform brightness.

It may be noticed that the two bright stars lying outside the limits of Table IV. are β Orionis and β Tauri. For the latter the formula gives $(m = 1{\cdot}8)\varpi' = 0''{\cdot}0630$, which agrees closely with $0''{\cdot}0600$ given in Table I. On the other hand, the formula gives a parallax $0''{\cdot}126$ for Rigel (mag. 0·3), which is very different from the value $0''{\cdot}0014$ actually found. The latter value is, however, confirmed by the Gill-Finlay parallax, and there is no reason to doubt that Rigel really is an exceptionally bright, distant star.

8. In the earlier paper in the *Monthly Notices* on the A type stars it was suggested that a component of velocity above 60 km. a second might be considered to exceed a reasonable limit. Such a limit was at best purely arbitrary, and there is definite reason for thinking that it is placed too low. For Mr. B. Boss has discovered (*A.J.*, 629) a whole group of stars which are moving relative to the Sun with a velocity of 95 km. a second. Hence we must be chary in rejecting results which appear at first sight improbable, and must in fact reconsider some of the results which were set aside on this ground. Owing to the position of the apex of Mr. Boss's group, by far the greater part of the velocity appears in the v-component. Hence it occurred to me to try whether three large v-components among the A type stars might not indicate membership of the group mentioned. Not only was the result successful, but it was found possible to include also two other stars which showed the same characteristic. And similarly among the present stars κ Cephei can be attributed to the same group. The test adopted is an extremely simple one. For let us assume that a

suspected star is moving with this particular velocity of 95 km./sec. relative to the Sun. Then its parallax ϖ is given by the equation:

$$95^2 = (V - K)^2 + \{(15\mu_\alpha \cos\delta)^2 + \mu_\delta^2\}\varpi_0^2/\varpi^2,$$

where $\log \varpi_0 = 0{\cdot}6758$. The components of velocity can then be calculated as before. The results are shown in Table V.

TABLE V.

P.G.C.	Star.	Type.	m.	m_0.	ϖ	u.	v.	w.	i.
264	θ Cassiop.	A5	4·5	4·8	″·0116	−13·3	+93·9	+ 6·3	2°·6
314	δ "	A5	2·8	3·7	·0153	−16·4	+93·6	− 0·3	2·5
3371	α^2 Can. Ven.	Ap	2·9	3·3	·0118	− 7·9	+93·9	+11·6	1·0
3626	α Draconis.	A	3·6	0·8	·0027	−18·6	+93·1	− 5·0	2·3
3825	δ Libræ.	A	4·8	2·8	·0039	−22·8	+92·0	− 6·6	4·6
5199	κ Cephei.	B9	4·4	0·3	·0015	−13·6	+94·0	− 3·2	0·3

9. There is little room for doubt that the motions of these stars are actually related. The mean components of velocity are

$$u = -15{\cdot}43, \quad v = +93{\cdot}42, \quad w = +0{\cdot}47,$$

giving for the position of the convergent

$$A = 99°{\cdot}4, \quad D = +0°{\cdot}3,$$

as compared with Mr. Boss's determination by Argelander's method

$$A = 99°{\cdot}15, \quad D = +0°{\cdot}5.$$

The closeness of the agreement is quite remarkable. The resultant motion derived from the mean components is 94·7 instead of 95 km./sec. as assumed as a starting-point. It is a duty to acknowledge here the value of Mr. Boss's work, the more so as it affords the means of accounting for precisely those results, or at least several of them, which appeared most anomalous in the present investigation.

It may, however, be thought that in the above change of method we are abandoning our primitive hypothesis of motions parallel to the Milky Way. That, of course, is in a sense true. But having obtained the motion of certain stars in another way, we can reverse the process and calculate the inclination of their motions, freed from the effect of the Sun's motion as assumed in § 2, to the plane of the Milky Way. These inclinations are given under i in the last column of Table V. So far from destroying the hypothesis in the sense in which I have always conceived it, these figures confirm its close approximation to the truth in a striking way. In none of the six cases does the inclination exceed 5°, and it is generally much less. The adjustment required by the parallaxes is not very great, and their order of magnitude remains unchanged. The six stars are, generally speaking, bright and

distant, and it will be noticed that the sequence in order of absolute magnitude m_0 is also the sequence of spectral type.

1c. Excluding four stars whose velocities appear exceptional, there remain in Table I. 26 stars whose motions form a fairly homogeneous set. These are too few to analyse by themselves with a view to the detection of groups with approximately common motions. They will be discussed with greater advantage in conjunction with the similar results deduced from the stars of type A. Pending the appearance of the *Lick Bulletin* in which these will be given in full, it seems better to defer the discussion of this question. But it may be said now that the tendency to form groups is quite well marked. We find no representatives of groups ii. (the Taurus group) or iv. (pp. 174, 175), but several of groups i. and iii. and a quite considerable proportion of stars which are possible members of a fairly large group which was not given with the others because it had been inadvertently divided into two parts, the practical identity of which thus escaped recognition. But a problem of critical importance presents itself at this point, because other investigators have been led to suppose that the mass of the B type stars reveal no tendency towards preferential directions of motion. The question therefore at precisely what point of spectral development such a tendency begins to manifest itself is one which demands the most careful examination, and for this reason any hasty conclusions on this subject are to be deprecated.

University Observatory, Oxford:
1912 *May* 6.

Measures of Southern Binary Stars in 1911. By John Tebbutt.

During the year 1911 I made the following measures of southern binary stars with the Cooke position and distance micrometer on the 8-inch equatorial.

Star	Fraction of Year.	Position Angle.	Distance.	No. of Evenings.
α Centauri	·580	216°·1	19″·12	5
β 416	·600	..	2·25	2
,,	·605	260°·2	...	4
γ Coronæ Aust.	·633	106°·5	2·28	5
γ Centaurii	·710	354°·9	1·62	6
β Muscæ	·750	356°·3	1·74	5

The above results are derived by weighting the results for each evening according to the number of measures and the conditions for observing. The conditions under which β Muscæ was measured were unsatisfactory. The observations above stated were made either in sunlight or in twilight.

Windsor, N.S. Wales:
1912 *March* 28.

The Position of the Sun's Axis as determined from photographs of the Sun, from 1874–1911, *measured at the Royal Observatory, Greenwich.* By F. W. Dyson, M.A., LL.D., F.R.S., and E. W. Maunder.

1. From 1874 to 1885 inclusive, the measures of positions and areas of Sunspots made at Greenwich were exhibited only in the form of a daily record of the Sun's surface. From 1886 onward, in addition to this daily record, the results of the measures were given in the form of a Ledger of Spot Groups; the various members of each group of spots being combined together for each day of observation, as shown in the specimen annexed. These ledgers, therefore, give the total area of each group for each day during which it remained visible; its mean longitude from Carrington's zero meridian; its mean latitude; and also its longitude from the meridian passing through the centre of the apparent disc. In 1907 ledgers on the same plan were published for the twelve earlier years, 1874–1885. Thus these sunspot ledgers give, in a form convenient for various investigations, the histories of about 7000 separate spot groups which have appeared on the Sun's disc since April 1874, when the Greenwich record commenced.

TABLE I.

Extract from the Ledgers of Groups of Sun-spots.

Group 1768. A large regular spot.

Date. Greenwich Civil Time.	Projected Area of		Area for Group		Mean Longitude of Group.	Mean Latitude of Group.	Longitude from Central Meridian.
	Umbra.	Whole Spot.	Umbra.	Whole Spot.			
1885. d					°	°	°
Sept. 21·443	0	65	0	197	23·9	+7·3	−81·2
22·441	15	150	20	201	23·2	+7·0	−68·6
23·428	9	120	8	106	23·1	+7·1	−55·7
24·574	0	154	0	101	23·0	+7·2	−40·7
25·407	65	354	37	202	23·3	+7·4	−29·4
26·447	45	432	23	224	23·3	+7·5	−15·7
27·132	95	440	48	224	22·8	+7·7	− 7·2
28·469	62	406	32	208	23·3	+7·8	+10·9
29·164	66	418	35	222	23·5	+7·5	+20·2
30·195	66	320	39	191	23·3	+7·7	+33·7
Oct. 1·388	20	246	15	189	23·4	+7·7	+49·6
2·171	33	206	32	202	23·5	+7·5	+60·0
3·381	0	61	0	118	23·3	+7·8	+75·7
Means ...	...	...	22	183	23·30	+7·48	...

2. In determining the heliographic latitudes of spots, the

position of the Sun's axis has been assumed as determined by Carrington,* viz.

$$I = 7^\circ 15', \text{ and } N = 73^\circ 40' \text{ for } 1850{\cdot}0,$$

where I denotes the inclination of the Sun's equator to the ecliptic, and N the longitude on the ecliptic of its ascending node. Changes in the apparent latitude of a spot group may arise from three causes:—

(i.) An error in this assumed position of the Sun's axis.
(ii.) An actual motion of the spot on the Sun's surface.
(iii.) Errors of observation or of measurement; particularly such as would introduce error in the determinations of the orientation of the photographs.

3. If I, the assumed inclination, requires a correction ΔI, and N, the assumed longitude of the ascending node, a correction ΔN, then, writing $y = \Delta I$, and $x = \Delta N \sin I$, the latitude, l, of a spot will require a correction

$$x \cos\theta - y \sin\theta,$$

where θ is the longitude of the group measured from the Sun's node. In the passage of a spot across the visible disc of the Sun, θ may in an extreme case increase by as much as 200°; and if the Sun's axis were alone in error, there would be a progressive alteration of the latitude given in the ledgers from which the quantities x and y could be determined. Examination shows that individual spots are affected by considerable proper motions, and it seemed very desirable to ascertain something of their character at the outset. More than 10 per cent. of the groups return to the visible hemisphere two or more times, and such recurrent groups, of which a catalogue was published in 1909 by Mrs. Walter Maunder, afford the means of examining their proper motions, free from any error in the assumed position of the Sun's axis. For the error of latitude, $x \cos\theta - y \sin\theta$, is the same when, in its second appearance, the spot group reaches, for the second time, the same longitude from the node.

4. *Examination of the Proper Motion of Recurrent Groups.*—The method of comparing the latitudes of a particular spot group in two successive apparitions was as follows. Carrington's assumed rotation period for the sun, 25·38 days, was corrected for the difference in the mean heliographic longitude of the group in the two successive rotations, and the nearest number of complete days so obtained was adopted as the standard interval between the dates that were to be compared. As experience has shown that Carrington's rotation period is somewhat in excess for the majority of spots, the interval thus obtained was usually 25 days. The mean latitude of the group in the first apparition was then compared with that in the second in every case where there were

* *Observations of the Spots on the Sun made at Redhill* by R. C. Carrington, p. 244.

pairs of observation separated by the standard interval, and the mean difference, Second apparition – First apparition, was deduced. In cases where a group returned three or more times, comparisons were made between the latitudes observed in different apparitions in all the ways available. Table II. gives a specimen of such a comparison.

TABLE II.

Group 3460 Mean Longitude 53°·44 Mean Latitude + 9°·79

,, 3492 ,, ,, 54 ·85 ,, ,, +11 ·90

Difference of longitude – 1°·41

,, in time – 0^{d}·10

Apparent sidereal rotation period, 25^{d}·38 – 0^{d}·10 = 25·^{d}28.

Interval adopted, 25 days.

Group 3460.		Group 3492		Difference of Latitude.
Date	Latitude.	Date	Latitude.	
1894.	°	1894.	°	°
March 28	+10·5	April 22	+11·9	+1·4
29	10·2	23	12·2	2·0
30	10·1	24	12·4	2·3
31	10·0	25	12·3	2·3
April 1	9·4	26	12·1	2·7
2	9·3	27	11·9	2·6
3	9·4	28	12·0	2·6
4	9·7	29	12·1	2·4
5	9·8	30	12·1	2·3
6	9·5	May 1	11·6	2·1
7	8·9	2	12·0	3·1
				Mean +2·35

The amount of material yielded by the 33 years, 1878–1910, for this discussion was as follows:—

TABLE III.

Seen in	Northern Groups.		Southern Groups.	
	No. of Groups.	Comparisons.	No. of Groups.	Comparisons.
2 rotations	230	230	245	245
3 ,,	53	106	69	138
4 ,,	12	36	19	57
5 ,,	3	12	8	32
6 ,,	0	0	1	5
7 ,,	1	6	0	0
Total	299	390	342	477

Or, in all, 641 groups yielding 867 sets of comparisons.

After this comparison had been made, the groups compared were catalogued in order of diminishing proper motion in latitude; motion in the northward direction being considered positive, and the northern and southern groups being catalogued separately. The results were then plotted down in a diagram to ascertain if there was any connection between the amount of proper motion of a group and its position in heliographic latitude, the number of groups observed in each degree of heliographic latitude being given for each quarter of a degree of proper motion, but no correlation appeared between the latitude and the direction or amount of the proper motion.

Similar diagrams were prepared in order to ascertain if any correlation could be detected between the amount of the proper motion and,

(i.) the mean heliographic longitude of the group,
(ii.) the year of observation,
(iii.) the interval after the date of the preceding minimum,
(iv.) the time in the year;

but in no case was any distinct evidence of connection obtained.

The mean amount of proper motion shown by the 230 northern groups seen only in two apparitions, amounted to $1^{\circ}\cdot20$ in a rotation; and for the 245 southern groups the corresponding amount was $1^{\circ}\cdot18$. There appeared, on the whole, a slight excess of motion in the northward direction, amounting in the mean to $5'\cdot5$ in a rotation for all spots. The northern spots, taken by themselves, gave a motion of $8'\cdot7$, the southern of $2'\cdot4$.

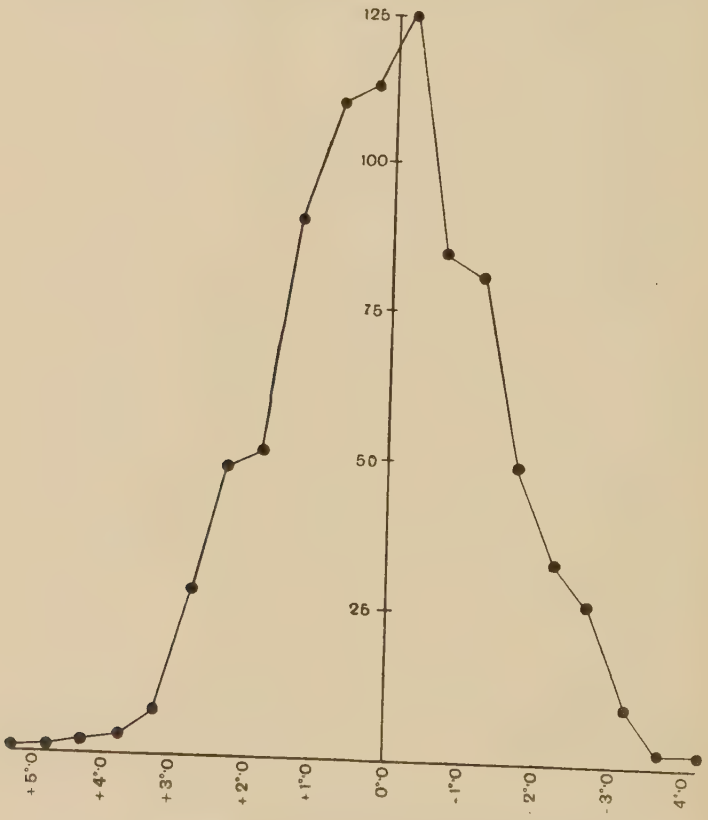
In the following table and diagram, the number of groups, north and south taken together, are given for each half-degree of proper motion. In this case all available comparisons have been used, and not merely those derived from groups seen only in two successive rotations.

TABLE IV.

Proper Motion in a Rotation.	No. of Observations.	Proper Motion in a Rotation.	No. of Observations.
From $+5^{\circ}\cdot5$ to $+5^{\circ}\cdot0$	1	From $+0^{\circ}\cdot5$ to $0^{\circ}\cdot0$	113
$+5^{\circ}\cdot0$ $+4^{\circ}\cdot5$	1	$0^{\circ}\cdot0$ $-0^{\circ}\cdot5$	125
$+4^{\circ}\cdot5$ $+4^{\circ}\cdot0$	2	$-0^{\circ}\cdot5$ $-1^{\circ}\cdot0$	85
$+4^{\circ}\cdot0$ $+3^{\circ}\cdot5$	3	$-1^{\circ}\cdot0$ $-1^{\circ}\cdot5$	81
$+3^{\circ}\cdot5$ $+3^{\circ}\cdot0$	7	$-1^{\circ}\cdot5$ $-2^{\circ}\cdot0$	51
$+3^{\circ}\cdot0$ $+2^{\circ}\cdot5$	27	$-2^{\circ}\cdot0$ $-2^{\circ}\cdot5$	33
$+2^{\circ}\cdot5$ $+2^{\circ}\cdot0$	48	$-2^{\circ}\cdot5$ $-3^{\circ}\cdot0$	26
$+2^{\circ}\cdot0$ $+1^{\circ}\cdot5$	51	$-3^{\circ}\cdot0$ $-3^{\circ}\cdot5$	9
$+1^{\circ}\cdot5$ $+1^{\circ}\cdot0$	90	$-3^{\circ}\cdot5$ $-4^{\circ}\cdot0$	2
$+1^{\circ}\cdot0$ $+0^{\circ}\cdot5$	110	$-4^{\circ}\cdot0$ $-4^{\circ}\cdot5$	2

One other possible correlation was sought for; namely, whether the proper motions in longitude showed any relation to those in

latitude. For this enquiry, it seemed desirable to take into consideration only those groups that consisted entirely or chiefly of a single well-defined stable spot: "regular" spots as they are termed in the Greenwich volumes. Not many instances of such groups could be found that were seen in successive rotations, 48 northern and 50 southern groups being all that were available. Inasmuch as the heliographic longitudes of the spots, as given in the



Proper Motion in Latitude of Recurrent Spot Groups.

Greenwich volumes, are computed from Carrington's uniform rotation period, 25·38 days, it appeared desirable that these should be corrected for the variation of the rotation period with latitude, and the formula derived by Mr. and Mrs. Walter Maunder * from the Greenwich sunspot measures, 1879–1901, was adopted; viz.

$$866'{\cdot}6 - 128' \sin^2 l.$$

* *M.N. R.A.S.*, vol. lxv. p. 823.

The resulting proper motion in longitude, after this correction had been applied, showed no correlation either with the heliographic longitude of the group or with its heliographic latitude. But a very marked tendency towards a retrograde proper motion became evident, as the following table (Table V.), will show. This readily explains itself as a characteristic of the life-history of spots of long duration. Regular spots usually form at the head of a stream, and in the early days of the development of the stream move rapidly forward in longitude. After the stream has attained its greatest dimensions and begun to diminish again, it is the following portion of the group that disappears the first, and the regular spot which was the original leader often remains alone, or with but a few very small companions, for a considerable time. In this stage of the history of the group, there is a strong tendency for the regular spot to move back again towards the longitude where it was originally formed.

This relation, however, is not one that affects the present enquiry, and it seems clear that, from the point of view of the determination of the Position of the Sun's axis, the proper motion in latitude of the spot groups may be considered as accidental.

TABLE V.

Proper Motion in Longitude of "Regular" Spots.

Northern Groups at an Interval of One Rotation.

First Group.			Second Group.			Diff. of Longitude II–I.	Diff. of Longitude Computed.	O.–C.
No.	Long.	Lat. +	No.	Long.	Lat. +			
	°	°		°	°	°	°	°
286	142·42	28·50	287	128·38	29·55	−14·04	−6·76	−7·28
321	88·99	25·95	327	84·02	24·37	− 4·97	−3·52	−1·45
342	323·47	11·93	351	327·31	10·18	+ 3·81	+4·82	−1·01
343	234·97	24·56	354	230·38	25·07	− 4·59	−3·26	−1·33
526	306·83	27·12	549	293·38	28·43	−13·45	−5·66	−7·79
553	263·30	20·43	575	257·30	20·76	− 6·00	−0·16	−5·84
575	257·30	20·76	591	262·15	21·02	+ 4·85	−0·34	+5·19
586	38·78	13·99	598	41·45	14·28	+ 2·67	+3·65	−0·98
691	221·78	− 0·48	718	224·81	+ 1·17	+ 3·03	+6·88	−3·85
722	160·78	20·78	749	155·95	20·91	− 4·83	−0·36	−4·47
744	176·69	23·08	767	168·39	21·19	− 8·30	−1·25	−7·05
797	292·54	14·43	812	296·73	15·28	+ 4·19	+3·19	+1·00
1217	200·06	1·92	1256	203·49	1·28	+ 3·43	+6·85	−3·42
1266	42·82	8·46	1301	41·46	10·42	− 1·36	+5·38	−6·74
1301	41·46	10·42	1339	45·18	9·53	+ 3·72	+5·20	−1·48
1324	180·39	7·48	1358	182·76	8·87	+ 2·37	+5·77	−3·40

TABLE V.—*continued.*

Northern Groups at an Interval of One Rotation—continued.

First Group.			Second Group.			Diff. of Longitude II–I.	Diff. of Longitude Computed.	O.–C.
No.	Long.	Lat.	No.	Long.	Lat.			
	°	°		°	°	°	°	°
1403	238·19	8·07	1420	239·65	9·29	+ 1·46	+5·62	−4·16
1856	199·60	5·36	1869	201·77	4·25	+ 2·17	+6·51	−4·34
2299	51·63	14·62	2318	53·52	14·15	+ 1·89	+3·41	−1·52
2405	148·84	18·60	2431	144·50	19·35	− 4·34	+0·86	−5·20
2448	105·93	9·49	2481	106·84	11·41	+ 0·91	+5·03	−4·12
2507	166·84	18·87	2545	163·14	18·38	− 3·70	+1·08	−4·78
2643	93·19	10·44	2675	91·78	10·60	− 1·41	+5·01	−6·42
2893	274·94	23·72	2930	268·79	24·87	− 6·15	−2·84	−3·31
3155	301·72	14·15	3203	300·63	12·81	1·09	+3·81	−4·90
3402	265·42	16·94	3443	263·99	16·80	− 1·43	+2·13	−3·56
3465	21·88	14·60	3495	19·68	15·53	− 2·20	+3·08	−5·28
3533	357·03	12·63	3581	359·63	12·75	+ 2·60	+4·17	−1·57
3642	300·93	18·38	3679	297·92	17·86	− 3·01	+1·39	−4·40
3702	42·00	16·05	3734	41·04	18·05	− 0·96	+2·04	−3·00
3732	56·26	13·31	3776	54·44	11·97	− 1·82	+4·19	−6·01
3916	308·79	13·04	3955	307·40	12·50	− 1·39	+4·14	−5·53
3933	183·49	13·93	3964	184·11	12·21	+ 0·62	+4·01	−3·39
4139	27·56	10·83	4180	26·11	11·48	− 1·45	+4·78	−6·23
5129	86·88	19·55	5146	84·07	18·08	− 2·81	+0·95	−3·76
5183	236·69	9·29	5204	244·83	6·56	+ 8·14	+5·55	+2·59
5912	201·36	17·93	5935	199·82	18·51	− 1·54	+1·34	−2·88
5931	361·20	21·97	5946	354·16	22·71	− 7·04	−1·39	−5·65
6045	322·62	13·84	6074	328·17	14·60	+ 5·55	+3·48	+2·07
6090	174·14	4·76	6115	176·51	5·74	+ 2·37	+6·44	−4·07
6134	356·74	12·71	6156	355·60	12·24	− 0·14	+4·26	−4·40
6172	76·58	7·13	6189	82·34	7·67	+ 5·76	+5·96	−0·20
6256	308·89	7·19	6280	310·60	7·46	+ 1·71	+5·98	−4·27
6436	331·48	11·42	6453	333·34	10·39	+ 1·86	+4·86	−3·02
6540	271·64	12·04	6558	271·71	12·78	+ 0·07	+4·30	−4·23
6577	167·94	12·08	6592	166·51	10·99	− 1·43	+4·64	−3·21
6592	166·51	10·99	6609	167·18	9·09	+ 0·57	+4·83	−4·26
6604	296·73	6·88	6621	298·56	9·66	+ 1·83	+5·74	−3·91

TABLE V.—*continued.*

Southern Groups at an Interval of One Rotation.

First Group.			Second Group.			Diff. of Longitude II-I.	Diff. o Longitude Computed.	O. – C.
No.	Long	Lat.	No.	Long.	Lat.			
		–			–			
	°	°		°	°	°	°	°
453	271·09	14·61	471	269·65	15·00	– 1·44	+3·21	–4·65
705	3·15	10·25	735	6·58	10·73	+ 3·43	+5·01	–1·58
846	50·18	21·65	868	43·15	22·38	– 7·03	–1·16	–5·87
927	213·94	17·93	946	211·04	19·45	– 2·90	+1·03	–3·93
1109	253·96	18·95	1126	249·48	18·68	– 4·48	+0·95	–5·43
1114	132·60	21·83	1133	128·78	22·41	– 3·82	–1·24	–2·58
1133	128·78	22·41	1169	118·59	23·30	–10·19	–1·76	–8·43
1144	35·28	9·78	1173	35·31	8·83	+ 0·03	+5·43	–5·40
1173	35·31	8·83	1198	34·49	9·72	– 0·82	+5·44	–6·26
1188	216·51	10·54	1213	218·89	12·96	+ 2·38	+4·56	–2·18
1198	34·49	9·72	1233	31·15	10·68	– 3·34	+5·12	–8·46
1255	208·19	8·54	1284	207·83	9·67	– 0·36	+5·50	–5·86
1284	207·83	9·67	1320	208·24	9·67	+ 0·41	+5·30	–4·89
1310	268·12	16·68	1350	268·09	17·07	– 0·03	+2·13	–2·16
1320	208·24	9·67	1357	207·97	9·33	– 0·27	+5·36	–5·63
1381	153·12	7·52	1409	152·27	6·60	– 0·85	+6·05	–6·90
1388	74·98	2·39	1410	80·04	2·34	+ 5·06	+6·79	+1·73
1393	28·53	9·32	1414	31·43	8·18	+ 2·90	+5·60	–2·70
1409	152·27	6·60	1425	151·29	7·25	– 0·98	+6·08	–7·06
1618	159·75	15·28	1636	158·54	13·65	– 1·21	+3·36	–4·57
1728	175·12	11·35	1748	173·17	13·93	– 1·95	+4·19	–6·14
1864	26·66	5·42	1876	32·17	6·51	+ 5·51	+6·31	–0·80
1876	32·17	6·51	1886	34·85	5·85	+ 2·68	+6·26	–3·58
1884	74·23	10·64	1897	74·10	11·69	– 0·13	+4·78	–4·91
1901	302·09	15·34	1910	300·53	13·28	– 1·56	+3·44	–5·00
1992	97·18	7·28	1998	97·53	7·49	+ 0·35	+5·96	–5·61
2108	334·50	7·45	2111	343·92	9·27	+ 9·42	+5·71	+3·71
2492	313·68	18·05	2528	313·09	17·06	– 0·59	+1·74	–2·33
2586	301·83	9·37	2623	303·22	6·99	+ 1·39	+5·77	–4·38
2612	91·18	22·04	2644	82·15	22·75	– 9·03	–1·44	–7·59
2923	331·63	14·66	2964	331·01	12·92	– 0·62	+3·68	–4·30
3002	9·45	15·93	3042	6·67	18·10	– 2·78	+2·06	–2·84
3005	321·64	12·54	3047	322·38	11·95	+ 0·74	+4·36	–3·62
3305	330·17	0·79	3352	335·93	2·03	+ 5·76	+6·86	–1·10
4049	81·92	17·88	4077	76·83	18·41	– 5·09	+1·38	–6·47
4395	123·99	20·55	4407	120·74	20·10	– 3·25	+0·01	–3·26

TABLE V.—*continued.*

Southern Groups at an Interval of One Rotation—continued.

First Group.			Second Group.			Diff. of Longitude II–I.	Diff. of Longitude Computed.	O. – C.
No.	Long.	Lat.	No.	Long.	Lat.			
	°	− °		°	− °	°	°	°
4579	270·38	9·10	4588	271·43	8·67	+ 1·05	+5·57	−4·52
4615	70·85	5·56	4634	72·97	6·18	+ 2·12	+6·32	−4·20
4687	100·56	7·09	4704	104·14	7·86	+ 3·58	+5·95	−2·37
4848	61·38	12·86	4855	61·21	12·68	− 0·17	+4·14	−4·31
4855	61·21	12·68	4857	61·15	12·84	− 0·06	+4·14	−4·20
5441	328·25	15·30	5471	329·36	16·76	+ 1·11	+2·56	−1·45
6324	143·21	4·99	6349	147·57	4·09	+ 4·36	+6·56	−2·20
6349	147·57	4·09	6361	153·94	2·08	+ 6·37	+6·74	−0·37
6420	162·25	14·86	6440	158·53	15·08	− 3·72	+3·13	−6·85
6441	144·39	3·01	6464	154·14	2·96	+ 9·75	+6·75	+3·00
6465	130·79	15·52	6489	127·16	16·32	− 3·63	+2·61	−6·24
6519	279·68	5·51	6539	279·00	5·29	− 0·68	+6·41	−7·09
6602	326·79	2·99	6618	331·98	2·98	+ 5·19	+6·75	−1·56
6771	182·60	14·06	6786	184·17	12·87	+ 1·57	+3·80	+2·23

5. *Method employed in determining the Position of the Sun's Axis.*—As in par. 3, let $x = \Delta N \sin I$; $y = \Delta I$; and let z be the latitude of any group of spots. To determine x, y, and z, we have from each spot group, a system of equations

$$x \cos \theta_1 + y \sin \theta_1 + z = l_1,$$
$$x \cos \theta_2 + y \sin \theta_2 + z = l_2,$$
$$\text{etc.}$$
$$x \cos \theta_k + y \sin \theta_k + z = l_k,$$

where l_1, l_2, etc., l_k, are the latitudes of the spot groups given for each day in the ledgers, and θ_1, θ_2, etc., θ_k are the corresponding longitudes measured from the node.

In these equations the angles θ_1, θ_2, etc., are approximately in arithmetical progression with a common difference whose mean value is about 14°·2.

Consider the simple case where θ has the thirteen values 0°, ± 15°, ± 30°, etc., ± 90°, which correspond sufficiently closely to a spot which is seen all the way across the disc. Let the probable error of each equation be ϵ. Solving by least squares, we obtain the normal equations:—

$$x\Sigma \cos^2\theta + y\Sigma \sin\theta \cos\theta + z\Sigma \cos\theta = \Sigma l \cos\theta \qquad p.e. \ \epsilon \sqrt{\Sigma \cos^2\theta}$$
$$x\Sigma \sin\theta \cos\theta + y\Sigma \sin^2\theta + z\Sigma \sin\theta = \Sigma l \sin\theta \qquad p.e. \ \epsilon \sqrt{\Sigma \sin^2\theta}$$
$$x\Sigma \cos\theta + y\Sigma \sin\theta + z \cdot 13 \quad = \Sigma l \qquad p.e. \ \epsilon \sqrt{13}$$

or
$$\left.\begin{aligned} 6x + 7\cdot64z &= \alpha \quad p.e.\ \epsilon\sqrt{6} \\ 7y &= \beta \quad p.e.\ \epsilon\sqrt{7} \\ 7\cdot64x + 13z &= \gamma \quad p.e.\ \epsilon\sqrt{13} \end{aligned}\right\}$$

Thus y is obtained with a probable error of $\frac{\epsilon}{\sqrt{7}}$, but x, from the way in which it is involved with z, with the much greater probable error, $2\cdot1\epsilon$. Thus nearly all the information as to the position of the Sun's axis is concentrated in one only of the normal equations. And in the general case, from each spot group, continued over a sufficient period, we obtain a single relation between x and y, with a considerable accuracy. Such an equation contains, within 3 or 4 per cent., all the information which this one spot group gives as to the position of the Sun's axis.

6. We next examine how nearly this equation can be obtained without having recourse to a least square solution. Still taking the ideal case where θ has the values $+90°, +75°, +60°$, etc., $-60°, -75°, -90°$, and writing the equations in order, we find by subtracting equation (13) from equation (1):—

$$y = \tfrac{1}{2}(l_1 - l_{13}) \text{ with } p.e.\ \frac{\epsilon}{\sqrt{2}},$$

and similarly

$$\cdot966y = \tfrac{1}{2}(l_2 - l_{12}) \quad ,, \quad \frac{\epsilon}{\sqrt{2}} \text{ from (2) and (12)}$$

$$\cdot866y = \tfrac{1}{2}(l_3 - l_{11}) \quad ,, \quad \frac{\epsilon}{\sqrt{2}} \quad ,, \quad (3) \quad ,, \quad (11)$$

$$\cdot707y = \tfrac{1}{2}(l_4 - l_{10}) \quad ,, \quad \frac{\epsilon}{\sqrt{2}} \quad ,, \quad (4) \quad ,, \quad (10)$$

$$\cdot500y = \tfrac{1}{2}(l_5 - l_9) \quad ,, \quad \frac{\epsilon}{\sqrt{2}} \quad ,, \quad (5) \quad ,, \quad (9).$$

etc.

Thus equations (1) and (13) give y with a *p.e.* of $\frac{\epsilon}{\sqrt{2}}$; equations (1) and (2) combined with (13) and (12) give $1\cdot97y$ with a *p.e.* of $\frac{\epsilon}{\sqrt{2}} \cdot \sqrt{2}$; equations (1), (2), and (3) combined with (13), (12), and (11) give $2\cdot832y$ with a *p.e.* of $\frac{\epsilon}{\sqrt{2}}\sqrt{3}$; and so on.

In this way y is determined

with *p.e.*	$\cdot707\epsilon$	using	one equation at each end.
,,	$\cdot509\epsilon$	,,	two equations ,,
,,	$\cdot432\epsilon$	,,	three ,, ,,
,,	$\cdot409\epsilon$	,,	four ,, ,,
,,	$\cdot392\epsilon$	,,	five ,, ,,
,,	$\cdot396\epsilon$	,,	six ,, ,,
and ,,	$\cdot378\epsilon$	,,	a least square solution.

Therefore, by combining the first five equations with the last five, a very close approximation is obtained to the accuracy of a least square solution, and by taking the first four with the last four, the *p.e.* is only increased from ·38ϵ to ·41ϵ.

If the change of longitude of a spot be taken as 14°·2, the probable errors, with which the displacement of the Sun's pole in a direction at right angles to the line joining it to the centre of the disc can be determined from spot groups extending over a given number of days, are given in the following table. The probable error of an observation of latitude given in the spot-ledgers for any day being taken as unity, the figures in the last column give the probable errors obtainable from a least square solution, and those in the columns headed 1, 2, 3, etc., the probable errors obtainable when 1, 2, 3, etc., equations at the ends of each group are subtracted from one another.

TABLE VI.

Extent of Group.	1.	2.	3.	4.	5.	6.	Least Square Solution.
14 days	·707	·506	·426	·388	·375	·378	·361
13 ,,	·712	·516	·441	·409	·402	·414	·388
12 ,,	·723	·532	·464	·439	·441	·467	·420
11 ,,	·748	·561	·498	·474	·497	...	·453
10 ,,	·787	·601	·546	·543	·580	...	·517
9 ,,	·845	·660	·615	·633	...	...	·587
8 ,,	·927	·744	·718	·733	...	...	·685
7 ,,	1·044	·867	·876	...	...	...	·820

From examination of this table it was decided for groups extending over 14, 13, or 12 days, to form an equation by subtracting the mean of the last 5 days from that of the mean of the first 5 days; for a group extending over 11 days to use 4 days at the beginning and 4 days at the end; and for groups extending over 10, 9, or 8 days to use 3 days at the beginning and 3 at the end.

It was further decided not to use groups extending over less than 8 days.

Thus for a group extending over 14 days, in which the longitudes from the node and latitudes are denoted by $\theta_1 l_1$, $\theta_2 l_2$ for each day of the group's appearance, the equation

$$\frac{x(\cos\theta_1+\cos\theta_2+\cdots+\cos\theta_5)}{5}+\frac{y(\sin\theta_1+\sin\theta_2+\cdots+\sin\theta_5)}{5}+\frac{l_1+l_2+\cdots+l_5}{5}$$
$$=\frac{x(\cos\theta_{10}+\cos\theta_{11}+\cdots+\cos\theta_{14})}{5}+\frac{y(\sin\theta_1+\sin\theta_2+\cdots+\sin\theta_{14})}{5}+\frac{l_{10}+l_{11}+\cdots+l_{14}}{5}$$

is formed. As θ_1, θ_2, etc., are approximately in arithmetical progression, an obvious simplification in the work is obtained by putting

$$\frac{\cos\theta_1+\cos\theta_2+\cdots+\cos\theta_5}{5}=\cos\frac{\theta_1+\theta_2+\cdots+\theta_5}{5}\cdot\frac{1+2\cos 14^\circ\cdot 2+2\cos 28^\circ\cdot 4}{5}$$

and similarly for the sines: *i.e.* the cosines and sines of the mean of the angles are taken and diminished by 6 per cent. Similarly, when 4 or 3 days are combined, the cosines and sines of the mean longitudes are diminished by 4 and 2 per cent. respectively.

7. The process actually adopted is shown in the following example:—

TABLE VII.

Spot Group 2496.

Date 1892	Long. of C.M. from Node.	Long. of Spot from C.M.		Latitude of Spot.	
d	°	°	°	°	°
April 27·52		−81·2		−23·8	
28·43		−71·8		−23·9	
29·40	145·4	−58·8	−58·0	−24·3	−24·16
30·41		−45·8		−24·4	
May 1·42		−32·4		−24·4	
2·23		−21·9		−24·3	
3·40		− 6·4		−24·7	
4·21		+ 3·8		−25·1	
5·35		+18·6		−25·6	
6·40		+31·6		−25·2	
7·42		+44·1		− 25·1	
8·50	154·1	+56·6	+55·9	−24·3	−24·56
9·43		+67·7		−24·3	
10·41		+79·7		−23·9	

Therefore for the mean of the days, April 27 to May 1, the longitude of the spot from the node $=145^\circ\cdot4-58^\circ\cdot0$, or $=87^\circ\cdot4$, and for the days, May 6 to May 10, $=154^\circ\cdot1+55^\circ\cdot9=210^\circ\cdot0$. We are led to the equations:—

$$\cdot94\{x\cos\ 87^\circ\cdot4-y\sin\ 87^\circ\cdot4\}-24^\circ\cdot16=+z$$
$$\cdot94\{x\cos 210^\circ\cdot0-y\sin 210^\circ\cdot0\}-24^\circ\cdot56=+z$$

which give by subtraction

$$0\cdot86x-1\cdot41y=-0^\circ\cdot40.$$

The longitude of the Central Meridian from the Node has uniformly been taken out for the Greenwich Civil Time $0^d\cdot4$.

The whole material available has been treated in this way, each group of spots yielding an equation of the above form, as shown in

the following table, which represents the year 1890, a year of few spots.

TABLE VIII.

No. of Group.	Mean Latitude.	No. of Days.	Equation.	Weight.
2111	− 9°·27	12	$+0°\cdot47x+1°\cdot38y=-0°\cdot68$	5
2117	+32·91	11	$+1°\cdot43+0°\cdot02=+0°\cdot75$	4
2132	−23·88	8	$+0°\cdot15-1°\cdot06=+0°\cdot60$	3
2133	−21·13	8	$-0°\cdot98-0°\cdot69=+0°\cdot10$	3
2135	− 6·43	11	$-0°\cdot82-1°\cdot20=+1°\cdot05$	4
2136	+18·03	10	$-0°\cdot88-1°\cdot25=-1°\cdot00$	3
2141	+21·14	13	$-1°\cdot58-0°\cdot20=-0°\cdot32$	5
2142	−19·33	9	$-1°\cdot12+0°\cdot62=-0°\cdot27$	3
2145	+24·52	9	$-1°\cdot27-0°\cdot17=+1°\cdot47$	3
2151	+22·19	8	$-1°\cdot10+0°\cdot36=-0°\cdot06$	3
2156	−23·29	14	$-1°\cdot14+1°\cdot19=-0°\cdot36$	5
2158	−24·73	8	$-0°\cdot96+0°\cdot62=-0°\cdot36$	3
2159	+19·91	13	$-0°\cdot13+1°\cdot58=+0°\cdot02$	5
2162	−27·48	8	$+0°\cdot62+1°\cdot00=+0°\cdot50$	3

The weights given to the equations are in proportion to the number of observations used in deriving the mean longitudes and latitudes. Thus the weight 5 is given to the spots of 14, 13, or 12 days' duration, as the means of the first five and last five observations were taken in such cases; similarly, the weight 4 is given to the spots of 11 days' duration and the weight 3 to those of 10, 9, and 8 days.

The following are the numbers of groups of each grade of duration that are available:—

TABLE IX.

Spots observed for 8 days . . .	331
,, ,, 9 ,, ...	285
,, ,, 10 ,, ...	282
,, ,, 11 ,, ...	314
,, ,, 12 ,, ...	537
,, ,, 13 ,, ...	454
,, ,, 14 ,, ...	34
Total,	2237

In a few cases it has been found necessary to reject a group because of some sudden and violent change that had taken place within it—usually in the nature of a fresh formation, which caused a sudden change in the position of the apparent centre of the group. Similarly the first or last day of the record of a group has been omitted, if it was clear from the change in its longitude and area that only part of the group was within the visible hemisphere on that day. An example of each case follows:—

TABLE X.

Example of a Group of Spots rejected on account of Violent Change.

Group 1587.

A number of small spots irregularly scattered over a considerable area. The group becomes somewhat more scattered, and the spots decrease in area until they have passed the central meridian on February 7. The following part of the group then rapidly increases in size, and becomes more compact, the individual spots coalescing with one another until on February 11 the group consists of three large spots so close together as almost to form a single very large spot. These have broken up by February 13. The preceding part of the group has disappeared by February 11.

1885.								
Feb.	1·387	25	169	32	219	222·5	−14·7	−68·0
	2·457	39	294	32	248	222·3	−14·7	−54·2
	3·220	28	313	19	216	222·5	−14·2	−43·9
	4·447	30	335	18	194	221·0	−15·4	−29·3
	5·500	68	245	37	130	219·9	−16·0	−16·4
	6·173	49	243	26	124	220·4	−16·1	− 7·0
	7·499	85	646	44	335	216·2	−18·6	+ 6·1
	8·153	176	1142	94	604	215·2	−18·6	+13·7
	9·480	214	1019	127	605	214·9	−18·5	+30·8
	10·509	176	888	125	619	213·8	−18·7	+43·4
	11·191	91	809	75	661	213·7	−18·9	+52·3
	12·545	21	200	31	291	214·6	−19·0	+71·1
	13·288	0	57	0	200	217·6	−18·6	+83·7
Means	...	...	...	51	342	218·05	−17·08	...

TABLE XI.

Example of a Group of Spots for which the first and last days were rejected, as the Group was partly out of sight on those days.

Group 1501.

A large regular spot, *a*, followed by another large spot of irregular outline, *b*. Some small spots are seen between *a* and *b*. *b* has broken up by October 6 into a number of spots, which undergo frequent changes during the succeeding days.

1884.	d					°	°	°
Oct.	2·412	50	217	107	464	20·3	+14·4	−77·6
	3·459	152	790	192	995	15·0	+14·3	−69·0
	4·435	176	1080	151	958	15·1	+14·4	−56·1
	5·141	210	1328	155	973	15·4	+14·5	−46·5
	6·274	223	1414	133	846	15·3	+14·5	−31·6
	7·522	297	1699	156	896	15·7	+14·3	−14·7
	8·461	351	1811	179	919	15·7	+14·1	− 2·4
	9·481	191	1896	99	980	16·0	+14·4	+11·4
	10·293	195	1532	137	830	15·6	+13·8	+21·7
	11·229	188	1473	112	882	14·7	+13·8	+33·2
	12·186	125	1075	92	775	15·2	+13·5	+46·2
	13·442	53	641	56	667	14·1	+14·1	+61·8
	14·458	0	282	0	526	14·2	+14·3	+75·3
	15·192	0	73	0	207	10·2	+14·1	+80·9
Means	...	...	...	112	780	15·18	+14·18	...

Combining these equations year by year, except for years near the sun-spot minima, when three or four years are taken together, and solving, the following values are obtained for x and y :—

TABLE XII.

No.	Period.	No. of Observations.	Weight.	x.	y.
1	1874–1877	75	282	− 9″·8	+12″·4
2	1878–1880	51	208	− 1·1	− 4·3
3	1881	77	304	+ 2·4	− 9·2
4	1882	86	345	+ 9·1	− 9·7
5	1883	112	448	− 0·2	− 4·8
6	1884	114	460	+11·7	− 0·5
7	1885	86	379	+11·5	− 4·5
8	1886–1888	81	339	− 1·8	− 8·1
9	1889–1891	86	355	− 3·9	− 4·2
10	1892	132	541	+ 6·1	− 9·4
11	1893	173	694	+ 1·6	− 8·6
12	1894	158	644	+ 3·1	+ 0·4
13	1895	122	497	+ 5·7	− 9·7
14	1896	65	251	− 4·2	−11·8
15	1897	52	221	+ 8·2	−12·8
16	1898–1900	78	311	− 4·3	−13·5
17	1901–1903	51	203	+ 5·3	− 1·4
18	1904	73	288	+ 1·8	+ 5·6
19	1905*	55	220	+ 8·7	− 6·5
20	1906	102	407	− 5·1	− 2·9
21	1907	116	473	+ 0·9	+ 8·3
22	1908	95	372	− 6·9	− 4·0
23	1909	87	369	− 9·7	+ 2·0
24	1910–1911	53	217	− 3·7	+ 3·2
25	1878–1888	607	2484	+ 5·3	− 5·7
26	1889–1900	866	3535	+ 2·1	− 7·7
27	1901–1911	632	2552	− 2·0	+ 1·8
28	1878–1911	2105	8546	+ 1·7	− 4·4
29	1874–1911	2180	8828	+ 1·4	− 3·9
30	1905†	112	467	+18·5	−22·1

* Excluding the Dehra Dûn photographs after 1905 April 4.
† Including all the Dehra Dûn photographs.

8. *Possible Effect of Orientation Error.*—In view of the large amount of material which has been discussed and the consequent freedom from accidental error which may be expected in the result, it seemed desirable to examine briefly the possibility of systematic error.

It does not seem likely that any serious error is introduced by making use of spots all the way across the disc, and therefore making use of observations when the spots are seen strongly foreshortened. The effects of foreshortening, which might be of importance in questions connected with longitude, are in a direction perpendicular to the displacements from which the correction to the position of the axis are to be derived.

The error most likely to affect the result is anything systematic in the determination of orientation. The different sources from whence the photographs are derived—Greenwich, Dehra Dûn, Mauritius, Harvard College, Melbourne, Kodaikánal, and the Cape —tend to diffuse errors from this cause among the other accidental errors, especially in those parts of the year when the photographs measured are drawn fairly indiscriminately from two or more contributory observatories. But in the winter months of most years, the series has been made up almost entirely from photographs taken at Dehra Dûn, and in the summer months the Greenwich photographs have similarly been in the large majority.

The errors of orientation have been chiefly determined in the following ways:—

(i.) At Greenwich, in the years 1875–1892, the orientation of the wires was determined by direct visual observation of transits, of spots, or of the Sun's limb.

(ii.) From 1885–1911, the orientation was determined by exposing a plate twice to the Sun at an interval of about $1^m\ 40^s$, the clock being stopped and the telescope being firmly clamped.

Observations were taken in these ways once a month or more frequently, and the zero of orientation adopted from the result.

The first method involves an observation both rough and difficult, and it was ultimately discarded in favour of the second. The second method is also open to drawbacks, for if the second exposure is made by setting the drop slit a second time, there is a danger of a slight disturbance of the instrument; if at the object-glass, the aperture has to be cut down so much that the image of the Sun's limb loses its sharpness, and an uncertainty is introduced into the measurement of the position angles of the cusps of the overlapping images.

The following table gives the determination of the error of orientation, as made by both methods in the year 1888, and the values actually adopted:—

TABLE XIII.

Determined from Double-Image Photographs.		Determined from Transit-runs of Spots.		Adopted.	
Date.	Orientation Error.	Date.	Orientation Error.	Limiting Dates.	Orientation Error.
1887.	° ′	1887.	° ′	1887.	° ′
Nov. 28	+0 33				
		Dec. 2	+0 32		
		3	+0 23	Dec. 3	
Dec. 9	+0 27	9	+0 36		
1888.		1888.		1888.	
		Feb. 1	+0 27		+0 30
Mar. 1	+0 34				
29	+0 30	Mar. 29	+0 24		
				Apr. 14	
				Apr. 16	
May 25	+0 46	May 25	+0 49	May 25	+0 36
				June 3	
				16	+0 42
June 23	+0 31	June 23	+0 47	23	
		July 30	+0 53		
Aug. 3	+1 1				+0 48
3	+1 12			Aug. 3	
		Aug. 7	+0 44		
				Aug. 8	
				Sept. 9	+1 0
				Sept. 10	
Sept. 11	+0 57				
		Sept. 12	+0 50		+0 54
Oct. 8	+0 51	Oct. 8	+0 57		
				Oct. 27	

Errors of orientation may be introduced either by an error of position of the polar axis of the instrument, or by a want of perpendicularity, either as between the polar and declination axes, or as between the declination axis and the telescope. Experience with the Astrographic telescope at Greenwich, where these quantities were determined by trails of stars at different declinations, shows that these errors of construction may well amount to several minutes of arc. In the Astrographic telescope, the errors, before they were corrected, amounted to 5′·8′, and 6′·0 respectively; introducing into the orientation the errors 5′·8 sec δ, and 6′·0 tan δ. If such errors had actually occurred in a photoheliograph, that depending on the tangent of the declination would have produced

a change in the orientation ranging from $+2''.5$ at the summer to $-2''.5$ at the winter solstice.

In the earlier years, when one or other of the Dallmeyer photoheliographs were alone available for use, the practice of adopting the orientation error for comparatively short intervals serves to prevent a systematic effect arising from a cause of this nature.

Since the Thompson photoheliograph has been introduced and mounted on the 26-inch equatorial, the observations for orientation have usually been grouped together for a year at a time. There is reason to believe that these errors, as well as the error of the position of the polar axis, are very small. The following table gives, for the fourteen years, 1898–1911, the mean difference, A. – O., for each month between the observed and adopted orientation :—

TABLE XIV.

Month.	No. of Obsns.	Mean Diff.	Month.	No. of Obsns.	Mean Diff.
		′			′
January . .	21	−2·1	July . .	24	−2·3
February . .	20	+3·3	August . .	25	+1·6
March . . .	17	−1·5	September .	21	−1·9
April . . .	19	−0·7	October . .	16	−0·3
May . . .	18	−0·5	November .	20	+0·1
June . . .	17	−1·6	December .	23	+2·4

A small term with an annual period is indicated here, but we may conclude that as far as the Thompson photographs are concerned, the correction to the position of the Sun's axis from this cause would not exceed 1′ in magnitude, and would mainly apply to x.

As regards the photographs from Dehra Dûn which, after those taken at Greenwich, are much the most numerous, the orientation until 1905 was determined by rotating the camera until a spot in transit was found to move along the horizontal wire. The observation was repeated about once a month. No correction therefore has been applied in the case of the Dehra Dûn photographs to the orientation as indicated by the wires. The instrument was kept in careful adjustment for many years, but in 1905 a serious error was discovered, caused by a dislocation of the polar axis during the great earthquake of April 4. Errors due to this cause enter into the heliographic results for 1905, but were corrected in 1906 by the substitution of photographs from other observatories. Since the beginning of 1908, a second exposure has been made at Dehra Dûn upon every plate taken for measurement. These second images have been measured as a check against any serious want of adjustment, but the orientation of the wires has been accepted as correct.

9. The photographs of the period 1878–1911, during which the series was practically complete, give the following final mean

corrections to Carrington's values for the inclination of the Sun's equator to the ecliptic, and for the longitude of its ascending node :—

$$\Delta I = -4''.4, \qquad \Delta N \sin I = +1''.7,$$

or

$$I = 7^\circ\ 10'.6, \qquad N = 73^\circ\ 53'.6 \text{ for } 1850.0.$$

The elements assumed by Carrington in the reduction of his own observations were :—

$$I = 7^\circ\ 10', \qquad N = 74^\circ\ 30' \text{ for } 1854.0.$$

or nearly the values obtained by M. Laugier from the discussion of 29 series of observations made by himself, viz. :—

$$I = 7^\circ\ 9', \qquad N = 75^\circ\ 8' \text{ for } 1840.0.$$

As a large amount of material has now been accumulated, and as in the preparation of the present paper it has been brought into a form very convenient for further discussion, it is proposed to continue the investigation under the following heads :—

(*a*) First of all, the material available can be substantially increased by using all the recurrent groups which give sufficient observations before their disappearance at the west limb and after their re-appearance at the east limb, for equations to be formed of a similar character to those employed in the present paper. The number of new equations which would thus be formed would be about one-third of those which have been used in this paper but their individual weight will not be quite so great.

(*b*) The variations from year to year in the values of x and y, deduced above, may be partly due to instrumental error, but are obviously in the main attributable to the proper motions of the individual groups; for usually a large amount each year is contributed by a few groups showing considerable movement. It is hoped therefore to subject the proper motions of the spots to a more searching examination than has been possible in the present paper.

(*c*) It is also proposed to examine separately the winter observations when photographs from Dehra Dûn were chiefly employed, from those made in the summer, when the Greenwich photographs were the chief source of information.

(*d*) A further point for examination will be to ascertain whether the same correction to the position of the axis is given by spots in different zones of latitude.

(*e*) Lastly, it is proposed to treat separately the groups of different durations, and in the case of spots, seen only for eight, nine, or ten days, to divide them into three classes, according as they were seen nearer the east limb, nearer the west limb, or centrally on the disc.

Royal Observatory, Greenwich.
1912 *May* 9.

The Effect of Magnetism on the Rates of Chronometers and Watches. By S. Chapman, M.Sc., B.A., and T. Lewis.

(*Communicated by the Astronomer Royal.*)

(1) The effect of magnetic fields on chronometers and watches first attracted attention some years ago, when electrical engineering was in its infancy. Visitors to electrical works came away, to find that the time-keeping qualities of their watches had become seriously impaired. Such experiences have produced, in the minds of a large part of the public, a rather exaggerated fear of any and every electro-magnetic machine. Several years ago, when the principles of design of electrical machines were but little understood, continuous current machines with large stray magnetic fields were not uncommon; nowadays, for reasons of economy, these stray fields are reduced to a minimum. This circumstance, together with the increasing use of alternating current machinery, generally renders electrical machines comparatively harmless to time-keepers.

(2) Nevertheless, if a chronometer or watch *is* placed in a sufficiently strong magnetic field, some change of rate may be expected. In view of the increasing use of electrical machinery on board His Majesty's ships, and the importance of accurate time-keeping for navigation, a series of experiments has been made at the Royal Observatory, Greenwich, during the past year, to determine what these magnetic effects will be. These experiments were undertaken, in the first instance, at the suggestion of the Hydrographer, who in particular wished us to determine the intensity of the magnetic field which would produce a change of one second in the daily rate of a chronometer.

The Experiments.

(3) After some short preliminary experiments, the apparatus used throughout the tests consisted of the large bipolar electro-magnet of a disused dynamo, fed by current from the lighting circuit of the Observatory. The armature being absent, there was a large stray field. The intensity, at points along the line of symmetry of the magnet, was taken to vary inversely as the cube of the distance from the poles; at small distances from the magnet this relation, of course, is only a rough approximation to the truth, but no correction has been applied to the calculated intensities on this account.

Assuming the above relation between the intensity of the magnetic field and the distance from the magnet, it is only necessary to know the absolute intensity at one distance in order to be able to calculate the intensity at any distance which is not too small. Our procedure was to place the magnet so that the lines of force (crossing the line of symmetry at right angles) were horizontal, and directly opposed to the Earth's horizontal field.

The point where there was no resultant force on a compass needle was readily found, and we thus determined the distance from the magnet at which the field intensity was 0·18 C.G.S. units (the value of the intensity of the Earth's horizontal magnetic field at Greenwich). The current strength was noted at the same time, and occasionally throughout the experiments. It showed variations which at times amounted to 10 per cent. of the whole, but usually the variations were considered to be immaterial.

(4) The course of the experiments will now be briefly indicated, and the results summarised, in §§ 4–8. A discussion and theory of the effects will follow.

The chronometer tests were first undertaken, six chronometers, of various types and by different makers, being used. The effects of horizontal and vertical magnetic fields (in each case reinforcing the corresponding component of the Earth's magnetic field) were separately investigated. The tests were made with fields of different intensity; thus in any one test the six chronometers were all placed at the same distance from the magnet, but this distance was different in different tests. The change from vertical to horizontal fields was made, of course, by turning the magnet, the chronometers always being horizontal.

The procedure in each test was as follows: During a number of successive periods, each of two or three days' duration, the magnet was alternately in and out of action; the chronometers remained in the same position and were rated daily. Now, the daily rates vary not only because of the magnetic field, but also on account of temperature changes and other irregular causes. By taking the rates over a number of periods, however, during which the magnetism is alternately present and absent, the effect of these accidental causes can be eliminated, and the magnetic effect be clearly shown by the difference:—

Mean rate with the current on – Mean rate with the current off.

The lengths of the tests were naturally chosen with regard to the probable magnitudes of the magnetic effect and of the accidental variations of rate: thus as the strength of the field was increased, the duration of the tests was shortened.

At the outset we proceeded with great caution, increasing the intensity of the field by gradual steps in successive tests; we had no knowledge of the probable effect of a given field (say, one of intensity twice that of the Earth's horizontal magnetic field), and it was necessary to avoid any possible risk of injury to the chronometers. The intensity was increased till a change of one second in the daily rates of some of the chronometers was shown in the mean result of a sufficiently long test. The following table gives the observations in detail * for one of the tests, and will serve not

* We should like to record here our indebtedness to Mr. W. Bowyer, of the Observatory staff, for his help in making these observations, and more especially in those taken in the evenings.

only to illustrate the general plan of the experiments, but also to indicate the degree of accidental variability of the rates of the chronometers used.*

TABLE I.

Chronometer Tests in Magnetic Fields—Series III.

Horizontal Field ·83 + ·18 (*Earth's Horizontal Field*).

Temperature, °F.	Chronometer. Current	I. On.	I. Off.	II. On.	II. Off.	III. On.	III. Off.	IV. On.	IV. Off.	V. On.	V. Off.	VI. On.	VI. Off.
		s		s		s		s		s		s	
56		+1·0		+4·6		+1·8		−2·7		−1·3		−2·6	
58	Daily rates	+1·5		+4·8		+1·4		−2·8		−1·2		−2·7	
57	determined		+0·6		+4·8		+1·6		−2·8		−2·1		−2·7
55	at intervals of one day.		+0·7		+4·2		+1·9		−3·0		−2·4		−2·2
54			−0·3		+3·4		+1·3		−3·0		−2·1		−2·6
55		+1·0		+4·2		+1·4		−2·8		−1·6		−3·1	
58		+1·8		+4·4		+2·2		−2·8		−0·5		−2·5	
61		+2·0		+4·8		+1·5		−2·6		−0·7		−2·5	
60			+1·3		+4·2		+0·9		−2·7		−1·7		−3·0
59	Additional		+1·1		+3·7		+2·2		−3·1		−2·2		−2·9
59	horizontal magnetic		+0·8		+3·3		+1·9		−3·4		−2·8		−3·3
61	field	+2·0		+5·1		+2·1		−3·1		−1·5		−3·3	
63	·83 C.G.S. units	+2·3		+5·4		+1·8		−2·5		−0·7		−2·2	
62			+1·8		+5·1		+1·6		−2·4		−1·7		−2·9
63		+2·8		+5·4		+2·4		−2·3		−1·3		−2·1	
64		+2·6		+5·1		+2·4		−3·0		−1·1		−3·4	
62			+1·9		+4·5		+2·4		−2·8		−2·3		−3·4
59			+1·3		+4·4		+1·9		−2·7		−2·5		−3·9
Means	Current on	+1·89		+4·87		+1·89		−2·73		−1·10		−2·71	
	Current off	+1·02		+4·18		+1·74		−2·88		−2·20		−2·99	
Change in daily rate— Current on − Current off		+0·87		+0·69		+0·15		+0·15		+1·10		+0·18	

When first beginning the experiments we endeavoured to find whether the rate of a chronometer varies with its orientation relative to the Earth's horizontal field or to the field of the magnet. As the fields we were then using were so weak that (as it afterwards appeared) the effect in any position was hardly appreciable, no effect of the kind mentioned was observed. It was judged better to examine one effect at a time, so that all the other tests

* Throughout the paper a positive rate (+) denotes that the time-keeper is gaining, a negative rate (−) that it is losing.

with the six chronometers mentioned were made with the XII on their dial towards the magnetic north (the horizontal field of the magnet was parallel to the Earth's horizontal field at the point where the chronometers were placed). The following table gives a summary of the results of these tests, with horizontal and vertical fields respectively.

TABLE II.

Changes of daily rates of Chronometers in fields of different strength.

Duration of Test in Days.	Total Strength of Magnetic Field.	No. of Chronometer. I.	II.	III.	IV.	V.	VI.
		Horizontal Magnetic Fields.					
	C.G.S.	s	s	s	s	s	s
16 to 24	0·5	+0·5	0·0	+0·1	+0·3	+0·4	+0·2
17	0·7	+0·2	0·0	−0·3	0·0	+0·1	+0·4
18	1·0	+0·9	+0·7	+0·2	+0·2	+1·1	+0·3
15	1·6	+1·8	0·0	+0·9	+0·3	+2·2	+0·8
		Vertical Magnetic Fields.					
24	0·7	0·0	−0·3	+0·1	0·0	+0·3	−0·1
6	1·6	+0·2	+0·3	−0·1	−0·3	−0·7	−0·2
9	2·8	−0·1	−0·1	−0·4	−1·1	−0·8	−0·3
12	9·2	−0·7	+2·5	+0·6	−1·8	−0·9	−3·9

(5) After the completion of these tests, some experiments on watches were undertaken. These were similar in general plan to the tests on chronometers, except that horizontal fields only were used (since vertical fields were seen to be harmless in comparison with horizontal fields), and these fields were much more intense. This was advisable, for a watch, which is carried about from place to place, may be exposed for a short time to a field much stronger than any in which a chronometer would be placed. The results of these tests, and of those next mentioned, are summarised in Table III.

(6) Concurrently with the above tests on watches, experiments were made to determine how far the watches might be shielded from the effects of the magnetic fields by being enclosed in iron boxes. Boxes of two kinds were tried: firstly, a rectangular box of dimensions $6 \times 5 \times 4$ inches, and $\frac{1}{4}$ inch thick, sent to us by the Hydrographer; and afterwards a smaller cylindrical box, $3\frac{1}{2}$ inches diameter, $1\frac{1}{4}$ inch deep, and $\frac{3}{16}$ inch thick, made in the Observatory workshop. As the following tables show, these boxes were very successful in diminishing the changes of rate.

TABLE III.

Tests of Watches in Horizontal Magnetic Fields of various strengths, showing the shielding effect of iron boxes.

(*A*) *Rectangular Box.*

Usher and Cole 25314. Daily rates.			Dent 29766. Daily rates.		
Current off.	Current on.	Current on in Box.	Current off.	Current on.	Current on in box.
Strength of Magnetic Field, 2·7 C.G.S.					
s	s	s	s	s	s
−3·0	...	...	...	...	...
...	...	−1·8	...	...	...
−2·0	...	...	...	...	...
...	...	−3·3	...	...	...
...	+ 9·0	...	...	...	...
−2·3	...	...	...	...	...
...	+ 7·1	...	...	...	...
−2·8	...	...	...	...	...
−3·5	...	...	−4·6	...	...
...	...	−1·3	...	...	−0·4
...	+ 7·7	...	...	+10·9	...
−2·7	...	...	−0·3	...	...
...	...	−3·2	...	...	−1·2
−4·0	...	...	+0·6	...	...
...	+10·0	...	...	+10·0	...
−2·9	+ 8·4	−2·4	−1·4	+10·5	−0·8

(*B*) *Cylindrical Box.*

Pennington 9781.			Dent 29766.		
Strength of Magnetic Field, 1·6 C.G.S.					
−2·7	...	...	−1·7	...	...
...	− 5·4	...	...	+ 4·7	...
−3·0	...	...	−1·5	...	...
...	− 5·8	...	...	+ 3·9	...
−4·3	...	...	−1·7	...	...
−3·3	− 5·6	...	−1·6	+ 4·3	...

TABLE III.—*continued.*

(*B*) *Cylindrical Box*—continued.

Pennington 9781. Daily rates.			Dent 29766. Daily rates.		
Current off.	Current on.	Current on in Box.	Current off.	Current on.	Current on in Box.
Strength of Magnetic Field, 2·7 C.G.S.					
s	s	s	s	s	s
−3·1	...	...	−2·1	. .	...
...	−3·3	...	...	+11·3	...
...	...	−2·0	...	...	+1·0
−1·1	...	...	0·0	...	...
...	...	+0·8	...	...	+1·8
−2·3	...	...	−1·6	...	...
...	−1·0	...	...	+9·0	...
−0·7	...	...	−1·9	...	...
...	...	−1·1	...	...	−1·5
−0·9	...	...	−4·8	...	...
−2·3	−2·2	−0·4	−1·9	+10·2	+0·8
Strength of Magnetic Field, 5·1 C.G.S.					
−3·3	...	...	−3·9	...	...
...	−18·1	...	...	+27·7	...
+1·4	...	...	−5·8	...	...
...	−21·8	...	...	+26·1	...
−0·8	...	...	−3·1	...	...
...	−15·5	...	...	+40·0	...
−2·2	...	...	−2·6	...	...
...	...	−3·8	...	...	−1·4
...	...	−4·0	...	...	−0·1
−2·7	...	...	−1·7	...	...
−1·5	−18·5	−3·9	−3·4	+31·3	−0·8

The above rates are derived from runs of one or more days.

(7) Finally, we returned to investigate how far the change of rate produced in a time-keeper depends on its orientation with respect to the magnetic field. These experiments led to some extremely interesting results. It was found that the change of rate (either in a watch or chronometer) was the same whether the VI or the XII on the dial pointed to the north (the magnetic

field being supposed to run north and south), and was *reversed in sign*, but practically unchanged in amount, when the III or IX was towards the north.* This suggested that in the intermediate positions, when the XII was towards the N.E., N.W., S.E., or S.W., no change of rate would be produced, and this was actually verified. Thus it appeared that a watch or chronometer may be placed in a magnetic field without any change of rate, provided that the field passes through the timekeeper in any one of four special directions.

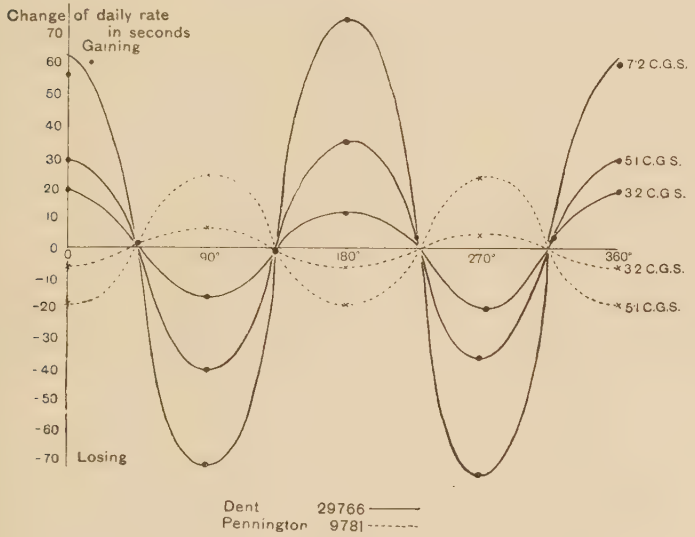


FIG. 1.

The following tables, and fig. 1, summarise and illustrate the results of these tests.

* In any one of these positions, however, different watches or chronometers behaved differently—one would go faster, another slower, as the result of the magnetic action.

[TABLE.

TABLE IV.

Tests of Watches in Horizontal Magnetic Fields of various intensities; effect of changing the direction of the field relative to the watch.

Dent 29766.					Pennington 9781.				
Direction of Pendant.	Current on. Period of Rating.	Current on. Daily Rate.	Current off. Period of Rating.	Current off. Daily Rate.	Direction of Pendant.	Current on. Period of Rating.	Current on. Daily Rate.	Current off. Period of Rating.	Current off. Daily Rate.
			Strength of Field = 3·2 C.G.S.						
		s	9 days	− 2·4			s	9 days	− 3·8
S.	2 days	+12·2			S.	2 days	− 7·2		
W.	2 days	−20·0	5 days	− 1·7	W.	2 days	+ 4·4	5 days	− 3·0
E.	2 days	−16·8			E.	2 days	+ 6·9		
N.	1 day	+19·0	2 days	+ 1·0	N.	1 day	− 6·3	2 days	− 4·6
N.E.	2 days	+ 0·9			N.E.	2 days	− 0·8		
S.W.	2 days	+ 3·9			S.W.	2 days	− 2·0		
S.E.	1 day	− 0·5	2 days	− 0·2	S.E.	1 day	− 2·5	2 days	− 4·0
N.W.	2 days	+ 2·0			N.W.	2 days	− 0·2		
			Strength of Field = 5·1 C.G.S.						
		s	2 days	− 1·7			s	2 days	− 4·3
N.	2 days	+28·3	1 day	− 7·0	N.	2 days	−19·4	1 day	− 0·6
W.	2 days	−36·7	2 days	− 3·2	W.	2 days	+23·9	2 days	− 4·2
S.	2 days	+36·6	1 day	− 0·4	S.	2 days	−18·3	1 day	− 4·2
E.	2 days	−40·2			E.	2 days	+22·8		
			Strength of Field = 7·2 C.G.S.						
		s							
N.	4h	+56							
N.N.W.	18h	+60							
W.S.W.	4h	+59	21h	− 0·8					
E.S.E.	3h	−39	1h	0·0					
S.	2h	+79	13h	+ 2·0					
W.	2h	−75	1h	0·0					

The results for the Pennington 9781 in this case are not comparable with the above, and will be given in Table IX.; see § 7 for explanation.

TABLE V.

Tests on Chronometers in a Horizontal Magnetic Field; effect of changing the direction of the field relative to the chronometers.

Kullberg 6279.			Johannsen 4708.			Dent 51944.			Johannsen 4600.		
Direction of XII on dial.	Daily Rate. Current on.	Daily Rate. Current off.	Direction of XII on dial.	Daily Rate. Current on.	Daily Rate. Current off.	Direction of XII on dial.	Daily Rate. Current on.	Daily Rate. Current off.	Direction of XII on dial.	Daily Rate. Current on.	Daily Rate. Current off.
						Strength of Field = 2·1 C.G.S.					
	s	s		s	s		s	s		s	s
		−0·1			−0·9	S.	−0·4		S.	−1·4	
N.	+1·0		S.S.E.	−1·5		N.	−0·5		N.	−2·4	
E.	−0·8		E.N.E.	+0·5		S.	−0·5		S.	−4·0	
S.	+0·8		W.S.W.	+0·4		E.	−0·7		E.	0·0	
W.	−0·7		N.N.W.	−0·2		W.	−0·1		W.	+1·3	
S.E.	+1·1		E.S.E.	−0·3		N.	−0·4		N.	−3·4	
		0·0			−0·8	N.		−0·8	N.		−1·8
N.W.	+1·2		W.N.W.	−0·2		S.	−0·3		S.	−4·3	
S.W.	−0·7		S.S.W.	−0·4		E.	−0·6		E.	+0·8	
N.E.	−0·5		N.N.E.	+0·4		W.	−0·3		W.	+1·1	
N.N.E.	+0·8		N.E.	+0·1		W.		−0·5	W.		−2·0
S.S.W.	+1·1		S.W.	−0·1		W.		−0·5	W.		−1·3
		+0·2			−0·5	N.		−0·3	−0·5 N.		−1·5 −1·7
W.N.W.	−0·1		N.W.	−0·7		E.		−0·8	E.		−1·9
E.S.E.	−0·4		S.E.	−1·3		N.E.	−0·8		N.E.	−0·8	
S.S.E.	+1·4		S.	−1·0		S.W.	−1·1		S.W.	−1·8	
N.N.W.	+1·7		N.	+0·2		S.E.	−0·2		S.E.	−1·8	
E.N.E.	−0·6		E.	+0·4		N.W.	−0·3		N.W.	−1·3	
		+0·1				N.W.		−0·4	N.W.		−1·5
					−0·5						
W.S.W.	−1·2		W.	−0·1							
W.S.W.	−0·8		W.	−0·1							

In the final test of the last series in Table IV., just given, it will be noticed that the watch Dent 29766 behaved as might be expected from the earlier tests of the same series. The other watch, Pennington 9781, however, behaved quite differently, and the results are therefore given separately in Table IX. It will be noticed that the change of rate is there reversed on reversing the watch from XII N. to XII S., while the effect in the XII E. and XII W. positions, though in itself large, is small in comparison with that in the former positions.

(8) While the tests on ordinary watches were going on, we made some tests on "non-magnetic" watches. In these watches all those parts of the balance system, escapement, and hands, which

usually are, or may be, of steel, are made of some non-magnetic material (frequently a palladium alloy). The mainspring, pivots, and arbors, ratchet and keyless work remain of steel, as in an ordinary watch.

These were tested in magnetic fields far more intense than those used with the ordinary watches; with fields of intensity up to about 20 C.G.S. units the effects were doubtful or small. Later a non-magnetic watch was placed right between the poles of the electro-magnet;* an ordinary watch would immediately stop if placed in a field such as the one there existing (or even in one much weaker). The maximum effect on the watch in question was to change its daily rate by less than two minutes—a remarkable result, considering the intensity of the field.

These tests on non-magnetic watches are summarised in the following table:—

TABLE VI.

Tests of Non-magnetic Watches—daily rates.

Webb 9310. Current		Webb 9312. Current		Webb 9282. Current		Webb 9313. Current		Webb 9324. Current.	
on.	off.	on.	off.	on.	off.	on.	off.	on.	off.
Strength of magnetic field, 16 C.G.S.									
s		s		s	s	s	s	s	s
+3·9	s	−0·9	s	s	−2·0	s	+3·6	s	−2·0
	−0·6		−0·3	+1·0		+3·0		+0·4	
+0·2		−0·3		−0·9		+3·7		−0·4	
	−0·1		+0·3	−2·2		+3·8		−1·5	
					−3·0	+3·5		−2·5	
				+1·3					
				−1·1					

(9) The above results, and a few others still to be given, will be discussed in the following sections, where a theory of the effects will also be developed. Almost all these results were obtained before this theory suggested itself; the experiments described in § 7 showed that our earlier ideas were untenable.

Discussion and Theory.

(10) It is evident that the motions of any machine which is composed of magnetisable or electrically conducting materials must be affected by the presence of a magnetic field. The original field will be modified by the presence of the iron and steel, which become magnetised, and subject to mechanical forces. These will directly affect the motion of the moving parts, and will indirectly affect the frictional forces. In so far as the magnetisation of the moving parts varies, hysteresis losses may occur. The

* This watch was very kindly lent to us by its maker, Mr. A. W. Webb of Charlton, with full permission to subject it to the most stringent tests.

elasticity of the springs may be affected by their magnetisation. The moving magnetised parts will induce eddy currents in the fixed conducting parts, and the secondary magnetic fields so produced will retard the motion.

In addition to these effects depending on the presence of magnetisable matter, there will be others depending on the electrically conducting moving parts; eddy currents will be induced in them, and the action of the magnetic field upon these will be to produce mechanical forces retarding the motion.

In a watch or chronometer the moving parts which are, or may be, of steel are the mainspring, the balance spring, the balance arm, and part of the rim, the fusee-arbor and chain, parts of the escapement, the pivots, arbors, and pinions, and the hands. Several small parts, like the ratchets and screws, may also be of steel. Almost every part of the mechanism is electrically conducting.

(11) The effects mentioned in § 10, as exemplified in a watch or chronometer, will now be examined in turn, in order to determine which affords the most likely explanation of the observed phenomena. Many of the effects referred to are obscure in their working and difficult to calculate, so that both experimental data and theoretical reasoning will be used in our examination.

In the first place, it is certain, on many grounds, that when the intensity of magnetic field does not exceed 9 or 10 C.G.S. units, the induced magnetisation in a time-keeper is only temporary. This follows (*a*) from the known magnetic properties of steel; (*b*) it was tested by us directly with a delicate needle, which revealed no residual magnetism in the watches or chronometers after the current was cut off; (*c*) it is shown by the way in which the time-keepers *suffer no permanent change of rate* after exposure to such magnetic fields,* and, (*d*) lastly, by the fact that a reversal of the field does not affect the change of rate.

One consequence of this is that we can rule out the effect of hysteresis loss due to changes in the magnetisation of the moving parts, since this loss is always extremely small when the magnetisation is only temporary; and when the changes are either very quick (as in the balance and balance spring), or very slow (as in the mainspring), the hysteresis losses are almost non-existent.

We can also neglect any changes in the elasticity of the steel springs, because these changes are infinitesimal when the magnetisation is weak and temporary; the experiments also show the unimportance of this effect, for it is evident that it would be practically independent of the orientation of the time-keeper in the field, while the observed effects vary largely with the orientation. Hence changes of elasticity cannot account for the observed phenomena.

Since the electrically conducting properties of the mechanism

*This was a striking feature observed throughout the tests. From observations made on Dent 29766, in a magnetic field of strength 7·2 C.G.S., it appeared that the rate recovered its normal value in ¼ minute after the current was cut off.

are the same in a non-magnetic watch as in an ordinary watch, the effect of eddy currents in the balance system will be alike in the two cases, whereas the actual behaviour of the two types of watch is by no means the same. Thus eddy currents in the balance system cannot explain the peculiar effects of a magnetic field on ordinary watches. For the same reason these effects cannot be laid to the charge of increased friction in the mainspring, pivots, and arbors, which are the same in non-magnetic watches as in ordinary watches.

Further, since the secondary magnetic field due to the eddy currents which are induced in the frame plate, and other conducting parts, by the motion of the magnetised balance, is negligibly small compared with the primary magnetic field, it may be left out of consideration when we are dealing with effects depending on the direct action of the primary field on the magnetised parts of the watch.

No explanation of the observed effects can be sought for in connection with the escapement mechanism, the function of which is to regulate the supply of energy from the mainspring to the balance. The energy of the balance is thus maintained constant (or nearly so), the loss due to various small resistances being made good through the escapement at each oscillation. Any modification of the energy supply from the mainspring, during its transmission through the escapement, would have no further effect than to change the amplitude of oscillation of the balance slightly. Such a change would have an inappreciable effect on the rate where the balance spring is isochronous.

(12) This leaves only one possible explanation of the phenomena in question, as all the other magnetic effects described in § 10 have been found, on examination, to be inadequate to produce the observed changes of rate. These must be due to the direct action of the magnetic field on the temporarily magnetised steel in the arm and rim of the balance. The energy supply from the mainspring will be practically unaltered; the mainspring and any fixed steel in the time-keeper are important only in so far as they modify the resultant magnetic field in the neighbourhood of the balance. The wheel train and escapement are hardly affected by the magnetism, so that the only magnetic effect which we need examine is that due to the couple acting on the magnetised portions of the balance.

For the sake of simplicity we shall consider the action as taking place only on the steel arm of the balance. The effect of the rim can be allowed for approximately by supposing the length of the arm to be altered in a certain ratio. This ratio cannot be precisely determined, and will vary considerably in different balances owing to the different length of rim used; on account of the uncertainty in the value of μ (see § 15*a*), this is not very material to our calculations.

(13) Thus the problem is reduced to the consideration of the vibrations, in a magnetic field, of a magnetisable bar which

carries a non-magnetic load, and is controlled by a spring which we may suppose isochronous and unaffected by the magnetism. Before entering on the mathematical treatment of the problem, however, the effect will be illustrated in the simplest possible manner in one or two special cases, omitting all detailed considerations which would obscure the main principles.

We will first examine the case in which the watch or chronometer is placed so that the balance arm, when at rest, lies along the direction of the resultant magnetic field. If the steel arm, which becomes magnetised, is displaced, it experiences a couple tending to restore it to its original position, since a magnetic field always tends to allineate a magnet in its own direction. The magnetic field, therefore, helps the balance spring to control the balance, or, in effect, it strengthens the balance spring. The balance thus vibrates more quickly, and the time-keeper increases its rate.

Next, suppose the watch or chronometer is turned so that the balance arm rests at right angles to the magnetic field. In this position the arm is not magnetised at all, and its equilibrium is undisturbed; but as soon as the balance is set in vibration, the magnetic field tends to bring the arm in line with itself, and therefore opposes the action of the balance spring, which tends to re-set the arm transverse to the field. The magnetic effect is therefore equivalent to a weakening of the balance spring; the balance vibrates more slowly, and the time-keeper diminishes its rate.

Evidently, in some position intermediate between the above, the rate of the time-keeper will be unaffected by the magnetic field. It will be remembered that the change of rate did actually vary in this manner, as the orientation in the field is varied (as described in § 7).

In the above discussion we have tacitly assumed that the arc of vibration of the balance does not exceed 180°; in practice this is not often the case. The effect is modified, on this account, very considerably; we shall consider this point, therefore, in the course of the mathematical theory, to which we now proceed.

Theory of Temporary Magnetisation in a Horizontal Field.

(14) Let us suppose the balance arm to be of uniform cross section, its sectional area being A, and its length $2l$. Let the moment of inertia of the balance be I; then $I = Ml^2$ very approximately, if M is the mass of the balance.

We will denote the intensity of the magnetic field by F; α will denote the angle which is made by the balance arm, in its mean position during a vibration, with the direction of the magnetic field. The strength of the field is supposed to be not great enough to magnetise steel permanently. θ will denote the angular displacement of the balance arm from its mean position at any time t, θ being measured in the same direction as α. Thus at time t the balance arm makes an angle $\theta + \alpha$ with the direction

of the magnetic field. The total arc of vibration of the balance arm will be denoted by $2\theta_0$.

If μ is the magnetic permeability of the steel of the balance arm, under the action of the magnetic force this arm will become a magnet whose magnetic moment at time t can be shown to be *

$$\frac{A\mu F \cos(\theta+\alpha)}{4\pi} \cdot 2kl \quad . \quad . \quad . \quad (1)$$

where k is a numerical constant depending on the shape of the balance arm; its magnitude in ordinary cases is likely to be about 0·75.

Hence the mechanical couple, tending to bring the balance arm to a position parallel with the direction of the magnetic field, is

$$\frac{k}{\pi}\mu A l F^2 \sin(\theta+\alpha)\cos(\theta+\alpha) = C \sin(\theta+\alpha)\cos(\theta+\alpha). \quad (2)$$

where, for convenience, we have written

$$C = \frac{k}{\pi}\mu A l F^2 \quad . \quad . \quad . \quad . \quad (3)$$

Next, we consider the action of the balance spring, which we shall assume to be isochronous, this being sufficiently near the truth for our purpose. Therefore, when the balance is twisted through an angle ϕ from its position of rest, the spring will exert a couple, tending to bring it back, of amount $K\phi$, where K is the coefficient of elasticity of the spring.

When the balance is vibrating in a magnetic field, the mean position of the swing ($\theta = 0$) will in general not be the position in which the spring exerts no force; the magnetic force acting in this position requires a constant force to be exerted by the spring in the opposite direction. Thus the couple due to the spring, at time t, is not $-K\theta$, but

$$-K(\theta-\delta),$$

where δ is a small quantity which remains to be determined.

Thus, in the usual way, we obtain the following equation of motion of the balance:—

$$I\frac{d^2\theta}{dt^2} = -K(\theta-\delta) - \tfrac{1}{2}C \sin 2(\theta+\alpha) \quad . \quad . \quad (4)$$

The first integral of this equation is

$$I\left(\frac{d\theta}{dt}\right)^2 = -K\theta^2 + 2K\theta\delta + \tfrac{1}{2}C\cos 2(\theta+\alpha) + \text{constant}. \quad (5)$$

* Strictly μ should be replaced by $(\mu - 1)$, but, as μ is large in the cases we shall consider, the correction is insignificant.

Since θ_0 is the semi-amplitude, so that $\frac{d\theta}{dt}$ vanishes when $\theta = \pm\theta_0$, we find that

$$4K\theta_0\delta = C \sin 2\theta_0 \sin 2\alpha, \quad . \quad . \quad . \quad (6)$$

and that the value of the constant of integration is such that the first integral may be written

$$I\left(\frac{d\theta}{dt}\right)^2 = K(\theta_0^2 - \theta^2) - \tfrac{1}{2}C \cos 2\alpha(\cos 2\theta_0 - \cos 2\theta) + \tfrac{1}{2}C\frac{\theta}{\theta_0} \sin 2\theta_0 \sin 2\alpha - \tfrac{1}{2}C \sin 2\theta \sin 2\alpha$$

$$= K(\theta_0^2 - \theta^2)\left[1 - \tfrac{1}{2}\frac{C}{K} \cos 2\alpha\{(\cos 2\theta_0 - \cos 2\theta) - \frac{\theta}{\theta_0} \sin 2\theta_0 \tan 2\alpha + \sin 2\theta \tan 2\alpha\}\right] . \quad . \quad . \quad (7)$$

Now, when the observed changes of rate are expressed as percentage changes in the time of a single oscillation,* it becomes evident that the mechanical forces due to magnetic effects are very small in comparison with those due to the balance spring, so that C/K is a very small quantity whose square can be safely neglected. On this understanding we can easily integrate the above equation, and obtain the following expression for the time T of a complete oscillation (double swing) of the balance :—

$$T = 2\int_{-\theta_0}^{\theta_0} \sqrt{\frac{I}{K}} \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}\left[1 + \frac{C}{4K} \cdot \frac{1}{\theta_0^2 - \theta^2} \cdot \{(\cos 2\theta_0 - \cos 2\theta) \cos 2\alpha - \frac{\theta}{\theta_0} \sin 2\theta_0 \sin 2\alpha + \sin 2\theta \sin 2\alpha\}\right] . \quad . \quad . \quad (8)$$

On integration, the second and third terms in the curly bracket lead to a null result, so that we get the equation

$$T = 2\pi\sqrt{\frac{I}{K}}\left[1 + \frac{C}{4\pi K} \cdot \cos 2\alpha \cdot \int_0^{\theta_0} \frac{\cos 2\theta_0 - \cos 2\theta}{(\theta_0^2 - \theta^2)^{\frac{3}{2}}} d\theta\right] . \quad . \quad . \quad (9)$$

In the absence of the magnetic field the term containing C as a factor vanishes, so that, denoting by T_0 the time of the complete oscillation when there is no magnetic field, we have

$$T_0 = 2\pi\sqrt{\frac{I}{K}}, \quad . \quad . \quad . \quad (10)$$

and

$$T = T_0\left[1 - \frac{CT_0^2}{16\pi^3 I} \cdot \cos 2\alpha \cdot f(\theta_0)\right] \quad . \quad . \quad (11)$$

* Thus, a change of 10 seconds in the daily rate, which is extremely large for a chronometer, and considerable for a good watch, represents only about ·01 per cent. change in the time of a single oscillation.

where we have eliminated the constant K in terms of I and T_0, and replaced

$$\int_0^{\theta_0} \frac{\cos 2\theta_0 - \cos 2\theta}{(\theta_0^2 - \theta^2)^{\frac{1}{2}}} d\theta \quad . \quad . \quad . \quad (12)$$

by $-f(\theta_0)$. The value of $f(\theta_0)$ can be readily calculated by quadratures; but this is unnecessary, for the Astronomer Royal kindly pointed out to us that

$$f(\theta_0) = +\frac{\pi}{\theta_0} J_1(2\theta_0),^{*} \quad . \quad . \quad . \quad (13)$$

in the usual notation of Bessel's functions. The function $f(\theta_0)$ was therefore calculated by means of a table of Bessel's functions. It is tabulated and plotted, over the range $\theta_0 = 0$ to $\theta_0 = 300$, in Table VII. and fig. 2 respectively.

The above formula for T may be more convenient if transformed into an expression for the actual change in the daily rate of a chronometer or watch; we thus get—

$$\text{Change of daily rate} = 60 \cdot 60 \cdot 24 \cdot \frac{C T_0^2}{16\pi^3 I} f(\theta_0) \cos 2\alpha \quad (14)$$

where the change in the daily rate is expressed in seconds, and is positive if the time-keeper goes faster, negative if it goes slower.

We will now substitute the values of C and I already obtained, in this last formula; we get

$$\text{Change of daily rate} = \frac{60 \cdot 60 \cdot 24}{16 \cdot \pi^4} \cdot k\mu T_0^2 \cdot \frac{A}{Ml} \cdot F^2 \cdot f(\theta_0) \cdot \cos 2\alpha \quad (15)$$

If we take 0·75 as the value of k, the expression becomes

Change of daily rate (in seconds)

$$= 41{\cdot}6 \cdot \mu \cdot T_0^2 \cdot \frac{A}{Ml} \cdot F^2 \cdot f(\theta_0) \cdot \cos 2\alpha \quad . \quad . \quad (16)$$

Here μ is the permeability of the steel of the balance arm,

* His proof is as follows: expanding $(\cos 2\theta_0 - \cos 2\theta)$ term by term, we have—

$$f(\theta_0) = -\int_0^{\theta_0} \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}} \left\{ -\frac{2^2}{2!} + \frac{2^4}{4!}(\theta_0^2 + \theta^2) - \frac{2^6}{6!}(\theta_0^4 + \theta_0^2\theta^2 + \theta^4) + \ldots \right\}.$$

Integrating term by term, and simplifying, it appears that—

$$f(\theta_0) = +\pi \left\{ 1 - \frac{\theta_0^2}{1!\,2!} + \frac{\theta_0^4}{2!\,3!} - \frac{\theta_0^6}{3!\,4!} + \ldots \right\},$$

and by comparing this with the power series for $J_1(2\theta_0)$, the stated relation becomes evident.

T_0 is the normal time of a complete oscillation (double swing) of the balance, M is the mass and l the radius of the balance, A is the sectional area of the balance arm, F is the intensity of the magnetic field, θ_0 is the semi-amplitude of the vibrations of the balance, $f(\theta_0)$ is a function of θ_0 which is tabulated in Table VII., and α is the angle between the direction of the field and the "set" (or position of rest) of the balance arm.*

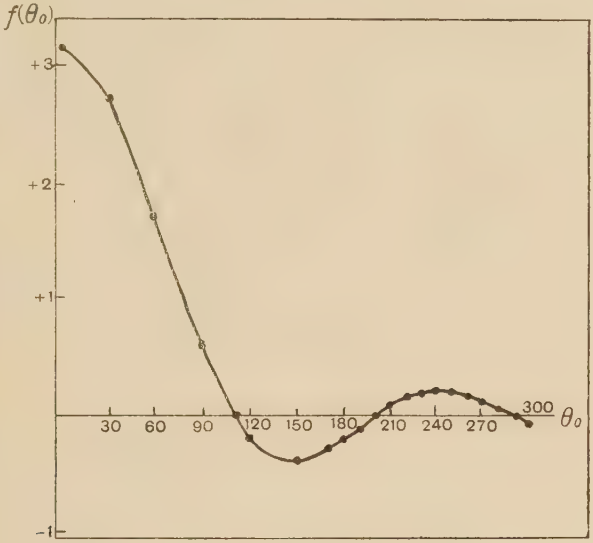


FIG. 2.—Graph of $f(\theta_0)$.

TABLE VII.

Values of $f(\theta_0)$.

θ_0.	$f(\theta_0)$.	θ_0.	$f(\theta_0)$.	θ_0.	$f(\theta_0)$.
0°	+3·14	180°	−·213	250°	+·193
30	+2·73	190	−·110	260	+·163
60	+1·70	200	−·009	270	+·118
90	+0·57	210	+·078	280	+·060
120	−0·20	220	+·145	290	+·010
150	−0·41	230	+·185	300	−·034
170	−0·31	240	+·202		

* More strictly, α is the angle between the direction of the field and the mean position of the balance arm during vibration; but the latter position is the same as the position of rest, if we neglect a small quantity of order C/K.

Discussion of the Formula.

(15) We will now discuss this formula, and compare it with the results of our experiments.

(*a*) It appears that the change of rate is directly proportional to μ—*i.e.* the effect is greater the more magnetisable the balance arm is. The value of μ depends greatly on the strength of the field, F. For weak fields μ is practically constant, for stronger fields it rapidly increases to a maximum, and, if the field is increased still further, μ tends asymptotically to zero. It varies considerably in different samples of steel, so that it is very difficult to get a good estimate of its probable value for such field strengths as we used in our experiments. It is likely, however, to be less than 100 for fields of intensity 1 or 2 C.G.S. units;* perhaps 40 or 50 is a probable value for $F=1$ or $F=2$, increasing up to 80 or 90 as F increases to 7 or 8. Throughout all our calculations we shall use C.G.S. units.

(*b*) Since the change of rate is proportional to T_0^2, it is seen that the slower the swing of the balance (and the relatively weaker the spring), the greater is the magnetic effect. In most modern chronometers the balance performs a complete oscillation in 1 second, so that $T_0=1$ generally. Occasionally $T_0=0{\cdot}8$ or $0{\cdot}9$.†

Influence of the Size and Weight of Balance.

(*c*) The next term in our expression for the change of rate is A/Ml. The effect therefore is increased by making the balance arm stouter, and diminished by making the radius larger and the balance heavier. An important consequence of this is that, in balances made of the same metal and similar in construction, the effect varies inversely as the square of the linear dimensions. This explains why watches are liable to be more affected by a magnetic field than chronometers are—the balances are fairly similar, but the linear dimensions are only about half as great, so that the effect on the rates should be three or four times as large in watches as it is in chronometers, with the same intensity of field and the same arc of vibration, etc.

One instance of this will suffice. We may refer to Table II. for the results of some tests on six chronometers, in a horizontal magnetic field of strength 1·6 C.G.S. units. In no case did the change of rate exceed 2·2 seconds; from Table III. (*B*) it will be seen that a watch rate was affected to the extent of 5·9 seconds in a similar field. As our numerical calculations in § 16 will show, neither figure represents the maximum possible effect (for

* In Bidwell's article on Magnetism in the *Encyclopædia Britannica* (11th ed.), vol. xviii. p. 344, a sample of steel wire is mentioned as having $\mu=86$ for $F=7{\cdot}50$, and $\mu=361$ for $F=20{\cdot}4$.

† Thus, in the case of the six chronometers used in the first two series of tests, the following are, in order, the values of T_0:—

·8, ·8, ·9, 1·0, ·8, 1·0.

a semi-amplitude exceeding 110°) in the corresponding case; but the illustration will serve.

Dependence of Change of Rate on Intensity of Field.

(*d*) The factor F^2 in our expression for the change of rate shows how rapidly the effect increases with the strength of the field, and since μ also increases with F, the rate of increase should be rather greater than that of F^2. To illustrate this point by our experimental data we will consider the behaviour of the watch Dent 29766, which was tested in five fields of different strength (see Tables III. and IV.). The maximum rates corresponding to these strengths are given below in Table VIII.; from these the mean value of the ratio

change of rate : (intensity of field)^2

has been found, and the changes of rate calculated back from this constant, in order to show how far the observations conform to the theoretical law.

Table VIII.

Variation of Change of Rate with Intensity of Field.

Intensity of field in C.G.S. units . .	1·6	2·7	3·2	5·1	7·2
Observed maximum change of rate . .	5·9	11	20	40	79
Calculated change of rate . . .	4·6	12	17	46	87

The agreement seems to be as good as could be expected, considering the number of accidental causes of variation which may act, and the degree of accuracy with which the intensity of the field is determined.

Effect of the Amplitude of Vibration of the Balance.

(*e*) We will next consider the influence of the amplitude ($2\theta_0$) of oscillation of the balance on the magnitude and sign of the change of rate. This enters into our expression for the change of rate through the factor $f(\theta_0)$, which is tabulated and plotted in Table VII. and fig. 2 respectively. The two most striking features are the rapid diminution in $f(\theta_0)$ as θ_0 increases, and the fact that $f(\theta_0)$ changes its sign as θ_0 passes through the angles 110°, 200°, 290°, and so on. Whereas we saw that a magnetic field will cause a watch to *gain* if the balance arm rests *along* the direction of the field, and to *lose* if the arm rests transverse to the direction of the field, when the semi-amplitude is less than 90°, it appears that this rule is reversed if the semi-amplitude of vibration of the balance lies between 110° and 200°. Thus two chronometers or watches of similar construction, placed in exactly the same position in a magnetic field, may be affected in exactly opposite ways if the semi-amplitude of vibration of balance of one is less than 110°, while in the other it is greater than 110°.

It so happens that the actual arcs in watches and chronometers lie fairly close to this critical value. In actual time-keepers θ_0 may lie between 90° and 270°, roughly, there being no general rule. In the case of any particular time-keeper which has a fusee, θ_0 may vary from time to time, from a number of causes, by about 10° or 20°. In a watch which has not got a fusee, the variation may amount to 40° or 50° (*e.g.*, 80° or 100° in the double swing), or even more, largely owing to the varying force of the mainspring. Thus it may easily happen, especially in the case of a watch without a fusee, that the magnetic effect on the rate may be in opposite directions at different times.

The considerations just adduced suffice to explain the different behaviour of the watches Dent 29766 and Pennington 9781, as described in Table IV. and illustrated by fig. 1. The difference must, in the main, be attributed to the difference in their running arc. It is, however, a very difficult matter to determine the arc of vibration of the balance, owing to the rapidity of the motion, and we have not been able to verify our theory in this case by actual observation.

It is interesting to notice that if θ_0 is equal to one of its critical values, 110°, 200°, etc., for which $f(\theta_0) = 0$, no effect on the daily rate of a time-keeper (at any rate, no effect of the first order, such as we have been considering) will be produced, however intense may be the magnetic field, or however the time-keeper may be placed in the field. In fact, if the arc of vibration equals any one of certain special values, an ordinary watch will behave as if it were non-magnetic. But this fact has little practical importance, owing to the variability of the arc, as already explained. It is advisable, however, so far as magnetic effects are concerned, not to have too small an arc, as the magnetic effect mounts up very rapidly as θ_0 diminishes below 110°.

Effect of the Direction of the Magnetic Field.

(*f*) Finally, we must consider the effect of the orientation of the time-keeper relatively to the magnetic field. The last factor, $\cos 2\alpha$, in our expression for the change of daily rate, shows that the effect is unaltered by reversing the magnetic field (*i.e.* changing α by π; the same fact also appears from the factor F^2, which is unaltered by changing the sign of F). It also shows that the effect is reversed by turning the time-keeper through a right angle, and, generally, that the effect on the change of rate has a double harmonic period in α. Our experiments fully corroborate the theory on this point, as is shown by the Tables IV. and V. and fig. 1.

It appears that a time-keeper, if placed in any one of four orientations with respect to a magnetic field, will suffer no change of rate (beyond second order effects, as in a non-magnetic watch), however intense may be the field, or whatever may be the arc of vibration of the balance. This fact might be of practical

importance if it were necessary to place a chronometer in a situation where there was a magnetic field which was constant in direction, if not in amount. A compass needle would give the direction of the field, and the "neutral" directions of the chronometer can easily be determined. In this way the chronometer might be placed so as to be unaffected by the magnetic field, without any futher precautions.

It is of interest to determine whether these neutral directions vary much from one chronometer or watch to another. Indicating directions in the time-keeper by the hour marks on the dial, it appears to be a fairly general rule, both in watches and chronometers, that the mainspring lies in the direction of IX or X from the centre, the fusee at I or II, and the balance at VI. The balance arm in its mean position generally lies nearly parallel to the VI–XII line—sometimes a few degrees on one side, sometimes a few degrees on the other. In considering the neutral positions we must not lose sight of the fact that the mainspring, fusee, and smaller steel parts may modify the direction of the external field by a few degrees, in passing through the time-keeper. Hence, in general, we can say no more than that in a very large proportion of chronometers, and more especially of watches, the directions in which a magnetic field should pass through the time-keeper for maximum effect on the rate (positive or negative) are VI–XII and III–IX; and that in intermediate directions the effect falls away to zero.

Numerical Calculation.

(16) We will now proceed to apply our formula for the change of rate numerically, to compare its results with those of experiment.

In chronometers the semi-amplitude of vibration of the balance is rarely less than 100°, so that $f(\theta_0)$ will not exceed 0·31 * in the case of the majority of chronometers. It may be positive or negative, or zero; but it will not exceed the above value in numerical magnitude. Hence if we take $f(\theta_0) = 0{\cdot}31$ in our formula, we shall be getting an upper limit of the change of rate. We shall also take $f(\theta_0)$ as positive and $\cos 2\alpha$ as unity, in order to obtain the maximum change of rate produced by a given magnetic field, irrespective of sign.

A typical chronometer balance was taken, and the following measurements made:—

Weight	7 grams.
Radius	1·3 cm.
Section of balance arm	0·35 × 0·07 cm.

We shall take $\mu = 40$ (a probable value for fields of 1 or 2 C.G.S. units) and $T_0 = 1$ second. This completes all the data we need.

* See the table and curve on p. 599.

On substitution in our formula it appears that the maximum change of daily rate which may be expected in such a chronometer, due to a magnetic field of strength F C.G.S. units, is

$$1{\cdot}35\ \mathrm{F}^2 \text{ seconds.}$$

Putting F = 1, it will be seen that this value is in fair agreement with Table II.; no chronometer shows a change of rate exceeding this maximum value, though chronometers I. and V. approach the maximum. With F = 1·6, the maximum change of rate is calculated to be 3·5 seconds. For fields of intensity greater than 2 or 3 C.G.S. units, μ should be given a rather greater value than 40, as we saw in § 15 (α). Of course, with suitable values of the amplitude ($2\theta_0$), the magnetic effect on the rate of a chronometer may be less than the amount calculated above, or even zero (if $\theta_0 = 110$, etc.).

If we take F = 0·2, the maximum change is seen to be less than 0·06 seconds. Thus, since the Earth's horizontal field is 0·18 C.G.S. units at Greenwich, the maximum difference between the daily rates of a chronometer for different orientations in the Earth's field is less than twice this amount, or 0·1 second. Considering the accidental variations of chronometer rates, it is not surprising that we failed to detect this effect in our earlier experiments (see § 4). It appears, therefore, that the differential effect of the Earth's magnetic field for different positions of a chronometer is quite negligible.

By substituting the values of A, M, l, corresponding to watches, the maximum effect of a magnetic field in their case can similarly be obtained.

If we know the values of θ_0 and α in any particular case, the actual effect (and not merely the maximum effect) of a given magnetic field can, of course, be calculated.

Effect of Permanent Magnetism and Horizontal Field.

(17) So far the watch or chronometer has been supposed to be initially free from any permanent magnetism, and to have been placed in a magnetic field not strong enough to produce such. If, however, the watch has been permanently magnetised, the effect of a magnetic field (weak or strong) is considerably changed.

The modification of our theory to suit this case is easily made. Suppose that the balance arm has become a magnet whose magnetic moment in C.G.S. units is N. Then the mechanical couple acting upon it due to a field of strength F, when the angle between the arm and the direction of field is $\theta + \alpha$, is

$$\mathrm{NF} \sin(\theta + \alpha).$$

This takes the place of expression (2) in § 14—*i.e.*, we replace $\frac{1}{2}\mathrm{C} \sin 2(\theta + \alpha)$ by $\mathrm{NF} \sin(\theta + \alpha)$. Hence, as all the rest of the mathematics is unchanged, we can apply our former result in equation (14) if we substitute 2NF for C, and change θ_0 into

$\frac{1}{2}\theta_0$ and α into $\frac{1}{2}\alpha$. Thus we obtain the following formula for the change of daily rate, in seconds:—

$$60 \cdot 60 \cdot 24 \cdot \frac{NFT_0^2}{8\pi^3 I} f(\tfrac{1}{2}\theta_0) \cos\alpha = 348 \cdot \frac{N}{Ml^2} \cdot T_0^2 \cdot F \cdot f(\tfrac{1}{2}\theta_0) \cdot \cos\alpha.$$

It is not easy to compare the numerical magnitude of this expression with that of (14) or (16); it may, however, be remarked that the watch would not be permanently magnetised unless N were much greater than the value of (μFAl) which we were considering in § 14.

The fact that $\frac{1}{2}\theta_0$ is now the argument of the function f will tend to increase the effect of the permanent magnetism, since the function f increases rapidly as its argument diminishes. Also it will be seen that neither of the values of θ_0 (110° and 200°) which render the effect of temporary magnetism zero will do the same in the case of permanent magnetism; if $\theta_0 = 220$, $\frac{1}{2}\theta_0 = 110°$ and $f(\frac{1}{2}\theta_0) = 0$, so that some value of θ_0 between 200° and 220° is most suitable for diminishing effects of both kinds.

Again, since the effect of the direction of the field now appears in the form cos α, and not cos 2α, it is evident that a reversal of the watch in the field now reverses the effect on the rate. In intermediate positions the change of rate will fall to zero.

Effects of this kind were actually observed in the case of the watches Pennington 9781 and Dent 29766, which were permanently magnetised (the balance of the watch Dent 29766 was removed, placed between the poles of the magnet, and then replaced in the watch) in opposite directions, and placed in a field of strength 7·2 C.G.S. In this case there was a large change of rate even when the magnetic field was not present; this is due to the Earth's magnetic field. The results are given in Table IX.

The watches were demagnetised in the usual way by being placed in a rapidly alternating magnetic field, after which their rates became normal once more.

TABLE IX.

Effect on permanently magnetised Watches.

Pennington 9781.				Dent 29766.			
Duration of test.	Position of XII.	Current on.	Current off.	Duration of test.	Position of XII.	Current on.	Current off.
Strength of Magnetic Field, 7·2 C.G.S.							
hours			s	minutes			s
15	N.	s	+ 40	160	N.	s	− 940
22	N.	+ 1200		18	N.	− 3800	
4							
21	S.		− 60	84	S.		− 700
4	S.	− 1500		14	S.	+ 1800	
15	E.		− 25	250	E.		− 800
4	E.	+ 100		15	E.	− 500	
63	W.		− 35	12 hours	W.		− 800
7	W.	− 100		15 min.	W	− 1200	

Effect of Vertical Fields.

(18) If the foregoing theory of the effect of a horizontal magnetic field on the rate of a chronometer is correct, it is evident that a magnetic field which traverses the balance vertically (*i.e.*, perpendicular to the plane of vibration) should have no effect. We are inclined, therefore, to attribute the very small changes of rate which *were* observed in "vertical" fields, to the fact that the fields were *not* quite vertical in traversing the balance. This might well arise either from the original field of the magnet, which it was difficult to obtain quite vertical over the space occupied by six chronometers: or, again, from the effect of the mainspring in deflecting the field as it passed through the chronometer. The fact that the balance and mainspring are not on the same level would favour the production of such a deflection. It will be noted that a definite effect of a vertical field was found only when the field was fairly large: a small horizontal component of such fields would easily explain the observed changes of rate.

Magnetic Shielding by Iron Boxes.

(19) A few remarks on the best conditions for magnetic shielding may not be out of place. It can be proved mathematically that if a spherical shell, made of a highly magnetisable material like soft iron, is placed in a uniform magnetic field, then there will be a uniform, but much weaker, magnetic field inside the sphere. The reduction is very approximately proportional to the permeability of the iron (the quantity μ mentioned in § 14) and to the ratio of the volume of the iron to the volume of the whole space enclosed by the shell. Though the mathematical theory for hollow boxes of other shapes cannot usually be worked out, the general conclusions are likely to be the same. Hence the requirements for good shielding are that the iron should be very permeable (the value of μ is large for pure soft iron), and that the thickness should be as great as is conveniently possible.

Non-magnetic Watches.

(20) A detailed discussion of the causes of the changes of rate in non-magnetic watches would be difficult and perhaps uninteresting. The results of our tests show conclusively that the replacement of certain steel parts by non-magnetic materials, such as palladium alloy, is almost entirely effective for the desired purpose. The changes of rate produced in such watches may possibly be referred to eddy currents in the moving conductors, and changes of elasticity of the mainspring. Such effects cannot possibly be eliminated while the latter remains of steel and the rest of the mechanism of conducting metal. The precise allocation of the separate parts of the total effect, to the two causes mentioned, would be difficult to make, and we have not attempted to do so.

Index and Summary.

§§ 1, 2. Introductory. The original object of the experiments was to ascertain that intensity of magnetic field which will produce a change of one second in the daily rate of a chronometer.

§ 3. Description of the apparatus used; method of determining the intensity of the magnetic field.

§ 4. Tests on six chronometers in horizontal and vertical magnetic fields of different intensities. Table I. gives a specimen of the detailed observations, Table II. a summary of all these tests. It was found that a field of intensity 1 C.G.S. unit may change a chronometer's daily rate by one second. The change of rate was only temporary, lasting only while the chronometers were in the magnetic field.

§§ 5, 6. Similar tests in horizontal magnetic fields, of greater intensities, made on watches. The shielding effect of iron boxes was also examined, and was very satisfactory. The results are given in Table III.

§ 7. Tests on chronometers and watches to determine the influence of the direction of the magnetic field, relative to the watch, on the change of rate. If the XII on the dial is changed from (say) north to east or west, the change of rate may be reversed in sign; there are intermediate positions in which the rate is quite unaffected. See Tables IV. and V., and fig. 1.

§ 8. The field intensity in the above tests did not exceed 7·2 C.G.S. units. "Non-magnetic" watches were tested in much more intense fields, up to 20 C.G.S. units, and, later, between the poles of the electro-magnet. The changes of rate were extremely small, and temporary. They did not follow the same laws as those observed in the preceding tests.

§§ 9–12. Discussion of the effect of the magnetic field on the separate parts of a chronometer or watch, and the influence on the rate. The magnetic field is supposed to be horizontal, and not strong enough to produce permanent magnetisation of the time-keeper's rate. It appears that the steel in the balance is the real seat of the observed effects.

§§ 13, 14. Theory of the effect of a magnetic field on a non-magnetised balance. A formula is derived which gives the change of the daily rate, in seconds, due to a given magnetic field.

§§ 15, 16. The formula is discussed, and compared with the results of the experiments.

§ 17. Modifications in the theory of the effect of a horizontal magnetic field, for the case when the time-keeper is permanently magnetised. The effects are considerably changed, and are illustrated by further experimental results given in Table IX.

§ 18. On the observed changes of rate in "vertical" fields.

§ 19. The best conditions for magnetic shielding of watches.

§ 20. Remarks on the changes of rate in non-magnetic watches.

Note.—Some further experiments have been made while this

paper was going through the press; they will be described in full in forthcoming numbers of the *Horological Journal*. Watches were tested in fields of intensity up to 90 C.G.S. with permanent magnetisation. The arc of vibration of the balance was found to be gradually reduced, as the field intensity increased, until it became so small that the watch stopped (at field intensity 90 C.G.S.). As the semi-arc was reduced through the values 200° and 110°, the rates changed sign, in accordance with fig. 2 (p. 599). The rate resumed its normal value when the watch was re-started in the absence of the field.

Publication of Heliocentric Places of Planets by the Nautical Almanac Office.

(*Communicated by P. H. Cowell.*)

On January 17 the Lords Commissioners of the Admiralty signified their approval of the recommendations of the conference held in Paris last October.

It becomes the duty of this office among other things to prepare heliocentric places of planets and to communicate them to other offices in convenient time for their respective almanacs.

It has been decided therefore to publish these heliocentric places in advance in the form of appendices to earlier almanacs.

The following scheme will be adhered to if possible :—

Planet.	Group of Years.	Year of Appendix.	Interval in Days between consecutive Places.
Venus	1916–1918	1915	1
Mars	1916–1918	1915	1
Jupiter	1916–1940	1915	4
Saturn	1916–1940	1915	8
Uranus	1916–1940	1915	20
Neptune	1916–1940	1915	40
Venus	1919–1940	1916	1
Mars	1919–1940	1917	1

The calculations for the above programme are already complete, except for some interpolation.

The calculation of Mercury is undertaken by the Berlin Rechen-Institut.

The heliocentric places will not be republished in the annual almanacs, and the noon ephemerides will be re-arranged accordingly. A reference will be given in the explanation of the annual almanacs to the place where the heliocentric places of the year are to be found.

On Librations in Planetary and Satellite Systems.

By Ernest W. Brown.

1. This paper consists of an attempt to find the limitations which must be imposed on the initial conditions in order that commensurable relations between the periods of two planets or two satellites may exist. The subject has been studied at various times by Tisserand, Callandreau, Brendel, Poincaré, Andoyer, and others. Their labours have not succeeded in explaining the gaps in the distribution of the asteroids. On the contrary, there seems to be no reason, on purely gravitational grounds, why asteroids with mean motions twice, three times . . . that of Jupiter should not exist. I have attacked the question from a slightly different point of view, in searching for the least deviation from a circular orbit when there is this exact commensurability.

For this purpose, the equations of motion are put into a form which permits of the retention of the short-period terms along with the critical terms. It is then easier to see the part played by the former in preventing the critical angles from oscillating through very small arcs. It became necessary to exhibit the conditions separating libration from non-libration. For the case $n = 2n'$, the results are shown in a figure giving all the circumstances, from one end of the scale where no motion of a periodic character exists, through the libration region up to the ordinary cases of long-period inequalities. I have then discussed the variations of the osculating eccentricity under these circumstances. It appears that even for the limiting cases the eccentricity cannot always be very small. Thus the orbit must at some time differ considerably from a circle, even if the body be started in the most favourable circumstances so as to have its mean motion twice that of Jupiter.

For the purposes of illustration the conditions obtained are applied to a few of the known asteroids near this libration region. The actual calculation of their orbits has not been my object. The case of the Trojan group of asteroids for which there is libration about the value $n = n'$ is only briefly treated, since I have discussed it more fully elsewhere. A first approximation to the ranges of oscillation of these bodies about the triangular equilibrium points is obtained.

The Equations of the Problem.

2. The disturbing planet, Jupiter of mass m', and the disturbed planet an asteroid of infinitesimal mass, are supposed to move in the same plane. Jupiter moves in an elliptic orbit of which the elements are n', e', ϵ', ϖ'. The mean distance a' is properly defined by the equation $n'^2a'^3 = \text{sum of the masses of the Sun and Jupiter}$; but it will be more convenient to neglect m' in this equation and put $n'^2a'^3 = \mu$, the mass of the Sun. The elements of the osculating elliptic orbit of the asteroid are n, e, ϵ, ϖ, and a, defined by $n^2a^3 = \mu$.

Let r, L be the polar co-ordinates of the asteroid; r', L', those of Jupiter. Then the disturbing function for the motion of the asteroid is

$$\mathrm{R}=\frac{m'}{\{r'^2-2rr'\cos(\mathrm{L}-\mathrm{L}')+r^2\}^{\frac{1}{2}}}-\frac{m'r\cos(\mathrm{L}-\mathrm{L}')}{r'^2} \quad . \quad (1)$$

The equations of motion are

$$\frac{d^2r}{dt^2}-r\left(\frac{d\mathrm{L}}{dt}\right)^2=-\frac{\mu}{r^2}+\frac{\partial \mathrm{R}}{\partial r}, \qquad \frac{d}{dt}\left(r^2\frac{d\mathrm{L}}{dt}\right)=\frac{\partial \mathrm{R}}{\partial \mathrm{L}}.$$

Since

$$\frac{d\mathrm{R}}{dt}=\frac{\partial \mathrm{R}}{\partial r}\frac{dr}{dt}+\frac{\partial \mathrm{R}}{\partial \mathrm{L}}\frac{d\mathrm{L}}{dt}-n'\frac{\partial \mathrm{R}}{\partial \mathrm{L}}-n'\frac{\partial \mathrm{R}}{\partial \varpi'},$$

in which equation the elliptic values of r', L' in terms of t are supposed to have been substituted in R, we deduce the integral

$$2\mathrm{R}=\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\mathrm{L}}{dt}\right)^2-2n'r^2\frac{d\mathrm{L}}{dt}-\frac{2\mu}{r}-2n'\int\frac{\partial \mathrm{R}}{\partial \varpi'}dt+2\mathrm{A}, \quad . \quad (2)$$

where A is an arbitrary constant.

In undisturbed motion,

$$\left(\frac{dr}{dt}\right)^2+r^2\left(\frac{d\mathrm{L}}{dt}\right)^2=\frac{2\mu}{r}-\frac{\mu}{a}, \qquad r^2\frac{d\mathrm{L}}{dt}=\sqrt{\mu a(1-e^2)}.$$

Since these equations also hold for osculating elements, we have, by their use in (2),

$$\frac{\mu}{2a}+n'\sqrt{\mu a(1-e^2)}=\mathrm{A}-\mathrm{R}-n'\int\frac{\partial \mathrm{R}}{\partial \varpi'}dt \quad . \quad . \quad (3)$$

The four equations for the variations of the elements are, if we put

$$nt+\epsilon=\int n\,dt+\epsilon_1,$$

and then drop the suffix of ϵ_1,

$$\left.\begin{aligned}
\frac{da}{dt}&=\frac{2na^2}{\mu}\frac{\partial \mathrm{R}}{\partial \epsilon}, \qquad \frac{d\varpi}{dt}=\frac{na}{\mu e}(1-e^2)^{\frac{1}{2}}\frac{\partial \mathrm{R}}{\partial e},\\
\frac{de}{dt}&=\frac{na(1-e^2)}{\mu e}\frac{\partial \mathrm{R}}{\partial \epsilon}-\frac{na(1-e^2)^{\frac{1}{2}}}{\mu e}\left(\frac{\partial \mathrm{R}}{\partial \epsilon}+\frac{\partial \mathrm{R}}{\partial \varpi}\right),\\
\frac{d\epsilon}{dt}&=-\frac{2na^2}{\mu}\frac{\partial \mathrm{R}}{\partial a}+\frac{na(1-e^2)^{\frac{1}{2}}}{\mu e}\left\{1-(1-e^2)^{\frac{1}{2}}\right\}\frac{\partial \mathrm{R}}{\partial e}.
\end{aligned}\right\} \quad (4)$$

The third of these equations may be written

$$e\frac{de}{dt}=\frac{na(1-e^2)}{\mu}\frac{\partial \mathrm{R}}{\partial \epsilon}-s\frac{na(1-e^2)^{\frac{1}{2}}}{\mu}\frac{\partial \mathrm{R}}{\partial \epsilon}+\frac{na(1-e^2)^{\frac{1}{2}}}{\mu}\left(s\frac{\partial \mathrm{R}}{\partial \epsilon}-\frac{\partial \mathrm{R}}{\partial \epsilon}-\frac{\partial \mathrm{R}}{\partial \varpi}\right),$$

where s is a constant to be defined later. Divide this equation by $na(1-e^2)^{\frac{1}{2}}/\mu$, and in the first two terms of its right-hand member substitute the value of $\partial R/\partial\epsilon$ derived from the first of equations (4). With the use of $\mu = n^2a^3$, we obtain an equation which may be integrated into the form

$$\sqrt{\mu a}(1-e^2) - s\sqrt{\mu a} = -\int\left\{(s-1)\frac{\partial R}{\partial\epsilon} - \frac{\partial R}{\partial\varpi} - \frac{\partial R}{\partial\varpi'}\right\}dt - \int\frac{\partial R}{\partial\varpi'}dt + B, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where B is an arbitrary constant.

The equations (3) and (5) are exact under the given limitations. Eliminating e from their left-hand members, we obtain

$$\frac{\mu}{2a} + sn'\sqrt{\mu a} = A - n'B - R + n'\int\left\{(s-1)\frac{\partial R}{\partial\epsilon} - \frac{\partial R}{\partial\varpi} - \frac{\partial R}{\partial\varpi'}\right\}dt. \quad (6)$$

3. The equations (5), (6) replace the first and third of equations (4). They may also be deduced from the well-known canonical formulæ. The variations of a are mainly determined by (6); when these have been found, those of e are obtained from (5).

The chief object of the transformation is the determination of those orbits whose periods are commensurable, or nearly so, with that of Jupiter. *The constant* s *is now defined to be the ratio of the periods when this ratio is a proper fraction or an integer.* If then we put

$$n = sn'(1+x) = n_0(1+x),$$

s will be an integer, or a proper fraction, and x will represent the deviation from an exact ratio. For the Trojan group of asteroids, $s=1$, $n_0=n'$. For asteroids whose periods are nearly half that of Jupiter, $s=2$, $n_0=2n'$. The quantity x carries not only the perturbations of n, but also the mean value of the difference of n/sn' from unity, so that the equation serves equally well for the whole group of asteroids near or within a libration region. As, however, we shall expand in powers of x, asteroids not near this region should not be included. We can, however, always suitably change the value of s, so that they are near some libration region.

Up to the present time no asteroid has been found for which $x=0$ when $s=2/1$, $3/1$, which values lie within the main stream, and there are notable deficiencies of asteroids for other values of s. But for $s=1$ we have the four asteroids of the Trojan group, none of which have $x=0$ at the present time, but which appear to be capable of taking this value at proper intervals. In the former cases it appears that the mean value of x is not quite zero in general, while in the latter it is so.

4. Define a_0 by means of the equation

$$\mu = a_0^3n_0^2 = a_0^3s^2n'^2.$$

Then if $|x| < 1$,

$$\frac{\mu}{2a} = \frac{\mu}{2a_0}(1+x)^{\frac{1}{2}} = \frac{\mu}{2a_0}\left(1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 - \ldots\right),$$

$$sn'\sqrt{\mu a} = sn'\sqrt{\mu a_0}(1+x)^{-\frac{1}{3}} = \frac{\mu'}{a_0}\left(1 - \frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \ldots\right).$$

If then we put

$$\frac{6a_0}{\mu}(A - n'B) - 9 = C,$$

and use the expansion in (6), we obtain

$$x^2 - \frac{8}{9}x^3 + \ldots = C - 6\frac{a_0}{\mu}R + 6\frac{a_0}{\mu}n'\int\left\{(s-1)\frac{\partial R}{\partial \epsilon} - \frac{\partial R}{\partial \varpi} - \frac{\partial R}{\partial \varpi'}\right\}dt. \quad (7)$$

Next, let $|e| < 1$. Then

$$(\sqrt{1-e^2} - s)\sqrt{\mu a}$$
$$= \sqrt{\mu a_0}\left(1 - s - \frac{1}{2}e^2 - \frac{1}{8}e^4 - \ldots\right)\left(1 - \frac{1}{3}x + \frac{2}{9}x^2 - \ldots\right).$$

Hence, if we put

$$-\frac{2B}{\sqrt{\mu a_0}} - 2(s-1) = E,$$

Equation (5) may be written

$$e^2 = E + \frac{2}{3}(s-1)x + 2\frac{a_0}{\mu}n_0\int\left\{(s-1)\frac{\partial R}{\partial \epsilon} - \frac{\partial R}{\partial \varpi} - \frac{\partial R}{\partial \varpi'}\right\}dt$$
$$+ 2\frac{a_0}{\mu}n_0\int\frac{\partial R}{\partial \varpi}dt - \frac{1}{4}e^4 - \frac{4}{9}x^2(s-1) + \frac{1}{3}xe^2 + \quad . \quad (8)$$

If x, e be sufficiently small, we may obviously neglect, in a first approximation, powers of x beyond x^2 in (7) and powers and products of e^2, x beyond the first in (8). In the discussion which follows, $|x|$ will be less than ·03 and e less than ·25, and the neglect of these higher powers will not affect the main argument.

5. When the usual expansion of R in terms of the elliptic elements is made, any term is of the form,

$$p\cos\{i(L_0 - L_0') - j(L_0 - \varpi) + k(L_0' - \varpi')\} \equiv p\cos\alpha,$$

where

$$L_0 = sn't + sn'\int x dt + \epsilon, \quad L_0' = n't + \epsilon',$$

and i, j, k are integral; p is a function of a, a', e, e'. It contains m' as a factor, and is divisible by $e^{|j|}$, $e'^{|k|}$.

The critical terms are defined to be those for which

$$s(i-j) - (i-k) = 0, \quad . \quad . \quad . \quad . \quad (9)$$

*the cases $i=j=k$ being included except when $i=j=k=0$.** Since s is the ratio of two given integers, there will always be such terms present.

For any term whatever,

$$(s-1)\frac{\partial R}{\partial \epsilon}-\frac{\partial R}{\partial \varpi}-\frac{\partial R}{\partial \varpi}=-\{s(i-j)-(i-k)\}p\sin\alpha\,.$$

The right-hand member vanishes whenever the values of i, j, k satisfy (9).

The left-hand member therefore contains no critical terms, and it may be integrated in a first approximation as if the elements in p, α were constant.

6. Let us examine the nature of the error committed when the integration is performed on this assumption. For brevity, put $\lambda=s(i-j)-(i-k)$ so that $\lambda=0$ for critical terms.

We have

$$\int\left\{(s-1)\frac{\partial R}{\partial \epsilon}-\frac{\partial R}{\partial \varpi}-\frac{\partial R}{\partial \varpi'}\right\}dt=-\Sigma\int\lambda p\sin\alpha dt$$

$$=\Sigma\left\{\frac{\lambda p}{\dot\alpha}\cos\alpha-\lambda\int\left(\frac{\dot p}{\dot\alpha}-\frac{p\ddot\alpha}{\dot\alpha^2}\right)\cos\alpha dt\,,\right.$$

by integration by parts. The first approximation to $\dot\alpha$ is $\lambda n'$, where λ is not zero; hence $\dot\alpha$ is not in general small, so that its presence as a divisor will not alter the order of magnitude of any coefficient. There will, of course, be terms for which λ is small, but these can occur only for very small values of p; I am only concerned here with the principal terms.

We know that p contains the factor m', and the equations of variations show that dp/dt will contain m'^2 as factor; d^2a/dt^2 contains m' as factor. The last term of the above equation therefore has m'^2 as a factor. Its order of magnitude can be lowered only through small divisors arising in the integration. Critical terms may arise by combinations of non-critical terms. It will be seen below that the divisors arising with critical terms have in general the factor $\sqrt{m'}$. Whenever this is the case, the last term may be neglected in a first approximation.

Hence, putting $da/dt=\lambda n'$, we have, in a first approximation,

$$n'\int\left\{(s-1)\frac{\partial R}{\partial \epsilon}-\frac{\partial R}{\partial \varpi}-\frac{\partial R}{\partial \varpi'}\right\}dt=\Sigma p\cos\alpha=R_s\,,\quad .\qquad(10)$$

where R_s consists of the non-critical or short-period terms of R.

7. Denote by R_0 the terms in R for which $i=j=k=0$, and by R_c the critical terms. The results of the last section, when sub

* This definition is adopted in order to avoid confusion with Gyldèn's "elementary" and "characteristic" terms.

stituted in (7), (8), give us, as a first approximation to the values of e, s, the equations

$$x^2 = C - \frac{6a_0}{\mu}(R_0 + R_c), \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$e^2 = E + \frac{2}{3}(s-1)x + 4\frac{a_0}{\mu}R_s + 4\frac{a_0}{\mu}\int n'\frac{\partial R}{\partial \varpi'}dt. \quad . \quad (12)$$

It will be noticed that since $R = R_0 + R_c + R_s$, the terms R_s have disappeared from (11), while the critical terms appear only in (12) through x and the last term. The latter contains both short-period and critical terms. The short-period terms may be treated as in No. 6, and may therefore be supposed to be included in R_s; they are divisible by e' and will in general be small compared with the complete value of R_s, which includes, amongst others, the short-period terms independent of e, e'. With this understanding we may treat the last term of (12) as if it contained only critical terms in the first approximation.

8. *The critical arguments.*—These are the arguments for which $s(i-j) = i-k$. If we put

$$\left.\begin{aligned}(s-1)\theta &= sn'\int x\,dt + \epsilon - s\epsilon' + \varpi(s-1)\\ (s-1)\theta' &= sn'\int x\,dt + \epsilon - s\epsilon' + \varpi'(s-1)\end{aligned}\right\} \quad . \quad (13)$$

the expression given in No. 5 for any argument shows that the critical arguments are all of the form

$$j\theta - k\theta',$$

where $(j-k)/(s-1)$ is an integer.

The equations (13) give

$$\left.\begin{aligned}(s-1)\frac{d\theta}{dt} &= sn' \,.\, x + \frac{d\epsilon}{dt} + (s-1)\frac{d\varpi}{dt},\\ (s-1)\frac{d\theta'}{dt} &= sn' \,.\, x + \frac{d\epsilon}{dt}\end{aligned}\right\} \quad . \quad (14)$$

Substituting for $d\epsilon/dt$, $d\varpi/dt$ from (4), we obtain the differential equations for θ, θ':

$$(s-1)\frac{d\theta}{dt} = sn' \,.\, x - \frac{2na^2}{\mu}\frac{\partial R}{\partial a} + \frac{na}{\mu e}\sqrt{1-e^2}\left\{s - \sqrt{1-e^2}\right\}\frac{\partial R}{\partial e}, \quad (15)$$

$$(s-1)\frac{d\theta'}{dt} = sn' \,.\, x - \frac{2na^2}{\mu}\frac{\partial R}{\partial a} + \frac{na}{\mu e}\sqrt{1-e^2}\left\{1 - \sqrt{1-e^2}\right\}\frac{\partial R}{\partial e}. \quad (15a)$$

The four equations (11), (12), (15), (15a) are now the equations of the problem, replacing the original equations (4).

9. *The case* $s = 1$. The above formulæ fail when $s = 1$. The critical terms in this case have the arguments

$$(i - j)(n' \int x dt + \epsilon - \epsilon') + j(\varpi - \varpi') .$$

We put

$$\theta = n' \int x dt + \epsilon - \epsilon' , \qquad \theta' = \varpi - \varpi' ,$$

so that the critical arguments are of the form $i_1\theta + j\theta'$, and the equations satisfied by θ, θ' are

$$\frac{d\theta}{dt} = n'x + \frac{d\epsilon}{dt}, \qquad \frac{d\theta'}{dt} = \frac{d\varpi}{dt}, \quad . \quad . \quad (16)$$

where $d\epsilon/dt$, $d\varpi/dt$ have the values given in (4).

10. The main object of this paper being to obtain approximate numerical criteria for the existence of librations rather than the discovery of equations which will facilitate the work of computation of the orbits, I shall not attempt to discuss the equations in their general form, even though the latter be limited to the first approximation. From this point of view it is possible to neglect classes of terms which will not seriously modify the results. In the first place, the maximum value of the eccentricity of Jupiter is small (·06), and it will be neglected in the principal discussion. This restriction is afterwards removed for the case $s = 2$. It will be shown that the principal critical term depending on e' may be accounted for by a slight change in e which permits one to deduce the numerical results for the case $e' \neq 0$ immediately from those obtained for $e' = 0$.

Two cases will be examined. The Trojan group, for which $s = 1$, has been treated with some detail in a former paper.* I there found the orbits by treating them as periodic orbits enclosing the triangular equilibrium points. Inasmuch as the results were originally indicated by the methods of this paper, it seems proper to give a brief outline of the application to this case.

The case $s = 2$ will be treated in detail. The work is practically confined to the two principal critical terms of R. As long as the eccentricity of the asteroid is not too large, the results will not be very seriously in error. For large eccentricities (those greater than ·25, for example) they will be much changed by the critical terms which depend on higher powers of the eccentricity. On the other hand, cases of large eccentricity are much more simple, because it then appears that the variations of e are generally small compared with its mean value e_0 and expansions in powers of $(e - e_0)/e_0$ are possible and numerically useful. This is not the case when e_0 is small.

The equations for the first approximation are the same whether the angles be librating or performing complete revolutions: nothing is said about the higher approximations. On the analogy of the lunar theory, where no secular terms due to the purely

* *Monthly Notices*, vol. lxxi. (1911) pp. 438–454.

elliptic motion of the disturbing body are allowed, we should expect that the introduction of another variable angle θ in addition to the two angles L_0, L_0' would eliminate the secular terms in eccentricity and perihelion. These secular terms are, as is well known, really periodic, the periods being of the order $2\pi\mu/n'm'$. In libration, the slow variations of the eccentricity and perihelion are still periodic, but the periods are of the order $(2\pi/n')(\mu/m'e^i)^{\frac{1}{2}}$, where the index i depends on s. These periods, however, become infinitely long at the limit between libration and non-libration.

The Case $s = 1$.

11. Put $e' = 0$, then $\partial R/\partial\varpi' = 0$. Since $a_0 = a'$ we can no longer expand R in powers of a_0/a', and the equations (7), (8) must be used. They give in the first approximation,

$$x^2 = C - 6\frac{a_0}{\mu}R - 6\frac{a_0}{\mu}n'\int\frac{\partial R}{\partial\varpi}dt,$$

$$e^2 = E - 4\frac{a_0}{\mu}n'\int\frac{\partial R}{\partial\varpi}dt.$$

Referring to the definition of θ in No. 9, we have

$$L - L' = \theta + \phi,$$

where ϕ is expansible in powers of e, is divisible by e, and contains sines of multiples of the angle $\int n dt + \epsilon - \varpi$, which is equal to $n't + \epsilon' - \varpi + \theta$. Also

$$r = a(1 + \phi_1) = a_0(1 - \tfrac{2}{3}x + \tfrac{5}{9}x^2 \ \ldots\ )(1 + \phi_1),$$

where ϕ_1 is like ϕ, but consists of cosines instead of sines. If the distance between the planets is always greater than some given constant and if e be sufficiently small, we can expand R in powers of $e\cos(n't + \epsilon' - \varpi)$, $e\sin(n't + \epsilon' - \varpi)$. It is evident, therefore, that every term in $\partial R/\partial\varpi$ will contain a non-zero multiple of $n't + \epsilon' - \varpi$, and will have no critical terms. These terms, moreover, are the only non-critical periodic terms in R. Hence

$$n'\int\frac{\partial R}{\partial\varpi}dt = R_s, \text{ where } R = R_0 + R_c + R_s,$$

in a first approximation. The equations for x, e become

$$x^2 = C - 6\frac{a_0}{\mu}(R_0 + R_c), \qquad e^2 = E - 4\frac{a_0}{\mu}R_s.$$

We have also from (16), (4),

$$\frac{d\theta}{dt} = n'x - \frac{2n_0a_0}{\mu}a_0\frac{\partial R}{\partial a_0} + \frac{n_0a_0}{2\mu}e\frac{\partial R}{\partial e},$$

where expansion has been made in powers of e, the lowest being alone retained.

By construction $R_0 + R_e$ is the value of R when $e = 0$ together with terms which depend on e^2, e^4, ... Let us neglect e^2 and higher powers. Suppose that C is sufficiently small for x to be able to take the value zero. Then x will be of the order of magnitude $\sqrt{m'/\mu}$ at most. The second and third terms in the expression for $d\theta/dt$ may therefore be neglected in comparison with the first term.

When $e = 0$ we have $r = a_0 = r' = a'$ in R. Also $L - L' = L_0 - L_0' = \theta$. Hence

$$\frac{a_0}{\mu}\left(R_0 + R_e\right) = \frac{a_0}{\mu}R = \frac{m'}{\mu}\left(\frac{a_0}{\sqrt{2a_0^2 - 2a_0^2\cos\theta}} - \frac{a_0^2\cos\theta}{a'^2}\right)$$

$$= \frac{m'}{\mu}\left(\frac{1}{2\sin\frac{1}{2}\theta} - \cos\theta\right),$$

and

$$\left(\frac{d\theta}{dt}\right)^2 = C - 6\frac{m'}{\mu}\left(\frac{1}{2\sin\frac{1}{2}\theta} - \cos\theta\right) \quad . \quad . \quad (17)$$

This equation, which is sufficient to find θ in terms of t, is the same as that on p. 448 of my paper referred to above. The discussion there given need not be repeated here.

12. Let us apply this result to the four asteroids of the Trojan group, in order to find the extent of their oscillations about the equilibrium points. We have

$$\frac{d\theta}{dt} = x = \frac{n}{n'} - 1, \qquad \theta = \epsilon - \epsilon',$$

when osculating elements at any epoch are used. With $\mu/m' = 1048$, the equation (17) then gives C.

The maximum separation from the equilibrium point is given by $d\theta/dt = 0$, and therefore by the values of θ which satisfy the equation

$$\frac{1}{\sin\frac{1}{2}\theta} - 2\cos\theta = \frac{\mu C}{3m'}.$$

The following table gives the results. The elements are taken from the *Berliner Astronomisches Jahrbuch* for 1913, the date of the epoch (but not of the osculation) being reduced to 1910·0:—

The Trojan Group of Asteroids. Epoch 1910·0.

Planet.	n.	x.	ϵ.	θ.	$\frac{\mu C}{3m}$.	Range of θ.
	″		°	°		° °
J	299·13	...	181·8	...	...	...
588	294·70	− ·0147	243·8	62·0	1·079	50 to 72
617	300·53	+ ·0047	121·4	299·6	1·008	297 to 303
624	292·58	− ·0218	227·4	45·6	1·348	41 to 87
659	300·79	+ ·0055	252·3	70·5	1·075	50 to 71

The periods of these oscillations are so long that many years must elapse before these results can be tested with any degree of certainty. The asteroid 624 is interesting with its range of 46°.* The eccentricities and inclinations will affect the results, but not to a great degree since they only enter into the period with their squares. In all cases the elongation from the equilibrium point towards Jupiter is less than the elongation on the other side. A somewhat greater effect will be produced by the inclusion of the higher powers of m. For these the more extended developments given in my paper referred to above may be used.

13. With the actual existence of a range so large as that of 624 it is not unreasonable to hope that other asteroids may be found which show a still greater range, and possibly one moving in a horse-shoe orbit enclosing both the triangular equilibrium points and the unstable equilibrium point in opposition to Jupiter. The limiting horse-shoe orbit is 23°·4 from Jupiter at its nearest approach to the planet. While such a body might be found anywhere in its orbit, the chances favour those places where θ is stationary, that is, near where $\theta = \pm 23°{\cdot}4$ or π; at these places it is moving with the same mean motion as Jupiter.

The Case $s = 2$.

14. This is the case of bodies having nearly twice the mean motion of Jupiter. We have from (13),

$$\theta = 2n' \int x dt + \epsilon_1 - 2\epsilon' + \varpi ,$$
$$\theta' = 2n' \int x dt + \epsilon_1 - 2\epsilon' + \varpi' .$$

The critical terms of lowest orders are therefore those which have the first powers of e, e' as factors. All critical arguments are of the form $j\theta - k\theta'$, where j, k are any integers, positive, negative, or one of them zero.

I first treat the case $e' = 0$ and limit the critical terms to a single one, that with argument θ and having e as a factor. *Libration is defined to be a case of motion in which $d\theta/dt$ can have the value zero.*

Referring back to (11), we see that in and near libration x is of the order $\sqrt{m'e/\mu}$. The second term of (15) may therefore be neglected in comparison with the first. The last term in this equation has the factor $m'/\mu e$. We cannot neglect this unless it is small compared with x—that is, unless e is large compared with $(m'/\mu)^{\frac{1}{3}} = \cdot 10$. This will not generally be the case; the term must be retained.

In equation (15a) the second and third terms may be neglected in comparison with x. But since θ' does not occur when $e' = 0$, this relation is not required. The last term of (12) disappears with e'.

* Mr. L. J. Linders (*Archiv för Mat., Ast. och Fys.*, So. Vet. Ak. i. Stockholm, Bd. 4, No. 20) finds a maximum elongation of 17°·3 on either side of the equilibrium position.

In (11) R_0 will reduce to a constant under the given limitations. Put

$$R_c = -\frac{\mu}{a_0} pe \cos\theta, \quad C - 6\frac{a_0}{\mu}R_0 = c, \quad E_s = 4\frac{a_0}{\mu}R_s.$$

The coefficient p is then a well-known function of a_0/a', that is, of $2^{-\frac{2}{3}}$, and it has, in addition, the factor m'/μ.

The equations (11), (12), (15) reduce to

$$\left.\begin{aligned} x^2 &= c + 6pe\cos\theta, \\ e^2 &= E + E_s + \frac{2}{3}x, \\ y = \frac{1}{2n'}\frac{d\theta}{dt} &= x - \frac{p}{e}\cos\theta, \end{aligned}\right\} \quad . \quad . \quad (18)$$

where y will be written for $d\theta/2n'dt$. The short-period terms have been retained in the second equation; it will be seen that they have an important influence on the results when we come to discuss the maxima and minima of e. The short-period terms in the third equation only come into consideration at the limits of libration, that is, when $y = 0$. They will be mentioned in this connection below.

15. *Numerical Values.*—With $\mu/m' = 1048$, $a_0/a' = 2^{-\frac{2}{3}}$, I find from the well-known formulæ for the expansion of the disturbing function,

$$p = \cdot 000716.$$

We have $R = R_0 + R_c + R_s$. The portion R_0 contains the terms which are independent of the angular elliptic elements, while R_c contains the critical terms only. It will be seen immediately that the total range of R is at least $\cdot 003$. With the value just given for p, it is evident that the variations of R_0, R_c are small compared with those of R. We can therefore treat E_s as having a range equal to that of $4a_0R/\mu$.

16. *The approximate range of* R.—We have

$$\frac{4a_0}{\mu}R = 4\frac{m'}{\mu}\left[\frac{a_0}{\{r'^2 - 2rr'\cos(L - L') + r^2\}^{\frac{1}{2}}} - \frac{a_0 r\cos(L - L')}{r'^2}\right].$$

Since $L - L'$ is nearly of period $2\pi/n'$, let us first find the maxima and minima of R with respect to $L - L'$ as if these were independent of r, r'. I find the three values:—

$$L - L' = 0 \quad , \quad \frac{4a_0R}{\mu} = 4\frac{m'}{\mu}\left(\frac{a_0}{r' - r} - \frac{a_0 r}{r'^2}\right);$$

$$L - L' = \pi \quad , \quad \frac{4a_0R}{\mu} = 4\frac{m'}{\mu}\left(\frac{a_0}{r' + r} + \frac{a_0 r}{r'^2}\right);$$

$$L - L' = \cos^{-1}\frac{r}{2r'} \quad , \quad \frac{4a_0R}{\mu} = 4\frac{m'}{\mu}\left(\frac{a_0}{r'} - \frac{a_0 r^2}{r'^3}\right).$$

First neglect the eccentricities and all perturbations. Then $r = a_0$, $r' = a' = 4^{\frac{1}{3}} a_0$. The three values become respectively

·00498 , ·00299 , ·00193.

The greatest variation is therefore ·00305.

When the eccentricities are not neglected, the range alters. The eccentricity and perihelion of Jupiter oscillate about mean values owing to the atttractions of the other planets: the eccentricity may attain the value ·06. In general, the perihelion of the asteroid must make complete revolutions when the angle θ librates, so that r, r' will take their maxima and minima independently of one another and of those of $\cos(L - L')$. The value of R for $L - L' = 0$ will be greatest when $r = a_0(1 + e)$, $r' = a'(1 - e')$. The value of R for $L - L' = \cos^{-1}(r/2r')$ is least when $r = a_0(1 + e)$, $r' = a'(1 + e')$.

The various values for the range which result from different values for e with $e' = ·06$ are shown in the following table:—

Range of $4\frac{a_0}{\mu}R$.

e	Max.	Min.	Range.
0	·0050	·0019	·0031
·05	·0055	·0019	·0036
·10	·0062	·0018	·0044
·15	·0070	·0018	·0052
·20	·0080	·0017	·0063
·25	·0094	·0017	·0077
·30	·0113	·0016	·0097

The minimum varies but little with changes in e or e'. The least value of the range is ·0031, and this value is independent of θ, e.

17. *Discussion of the equations* (18).—Returning now to these equations, let us neglect E_s for the present, or—what amounts to the same thing in the analysis—let us consider that E may receive a variable addition amounting to at least ·003; this addition is nearly independent of the variations of x, e, θ, y, although the latter depend on it.

Eliminating e, θ, we obtain an equation which may be written,

$$(x + \tfrac{3}{2}\mathrm{E})\left\{ 3(x + \tfrac{3}{2}\mathrm{E}) - 4(y + \tfrac{3}{4}\mathrm{E}) \right\} = \frac{9}{4}\mathrm{E}^2 - c\,. \qquad (19)$$

We have also

$$\frac{dx}{dt} = 3e\frac{de}{dt}, \quad \frac{de}{dt} = -pn_0 \sin\theta\,. \qquad (20)$$

If e, x be eliminated from (18), an algebraic relation between

$y(=d\theta/2n'dt)$ and $\cos\theta$ is obtained; this is expressed by an equation which is of the fourth degree in y and of the second degree in $\cos^2\theta$. By a quadrature we can obtain t in terms of θ. Owing to the limitations imposed on e, x—that they shall be real—it is easier to discuss the equations in the forms (18), (19), (20).

(i) Unless $c=\frac{9}{4}E^2$, e cannot vanish. For since x, y must be finite, e can only be zero if $\theta=\frac{1}{2}\pi$, and then $x^2=c$, $c=\frac{9}{4}E^2$. If this case be excluded, e must in any motion always retain the same sign. The change $\pi+\theta$ for θ being equivalent to a change of sign of e, we may consider e as always positive.

(ii) The variables e, x, y are always in a 1, 1, 1 correspondence. Owing to (19), the relation between x, y is that between the co-ordinates on an hyperbola with asymptotes $x+\frac{3}{2}E=0$, $3x-4y+\frac{3}{2}E=0$. Since $\frac{3}{2}e^2$, $=x+\frac{3}{2}E$, is never zero, the relation between x, y is confined to one branch of the hyperbola, which is that lying to the right of the vertical asymptote. If $\frac{9}{4}E^2>c$, the branch lies below the oblique asymptote, and if $\frac{9}{4}E^2<c$, it lies above it; in the excluded case, $\frac{9}{4}E^2=c$, the hyperbola reduces to two straight lines which are the common asymptotes to them all. Hence x, y are in a 1, 1 correspondence. Also, since the relation between e, x is that between the co-ordinates of a parabola and e retains the same sign, e, x are also in a 1, 1 correspondence.

(iii) In libratory motion y can take the value zero for some value of t. When $y=0$, we have

$$x=-E+\sqrt{E^2-\tfrac{1}{3}c}=x_0,\quad e=\left\{\tfrac{1}{3}E+\tfrac{2}{3}\sqrt{E^2-\tfrac{1}{3}c}\right\}^{\frac{1}{2}}=e_0, \qquad (21)$$

where x_0, e_0 are the values of x, e for $y=0$. Hence $E^2\geqq\frac{1}{3}c$. The positive sign must be attached to the radical $\sqrt{E^2-\frac{1}{3}c}$ unless $E>0$, $\frac{1}{3}E^2<c<\frac{4}{9}E^2$ simultaneously when both signs are admissible; the significance of this case will be mentioned below. I shall, in general, suppose that with libration $E^2>\frac{4}{9}c$, but all cases are included in fig. 1 below.

(iv) The maxima and minima of θ are given by $y=0$. We then have

$$p\cos\theta_0=e_0x_0, \qquad (22)$$

where e_0, x_0 have the values (21).

(v) The maxima and minima of e are, from (20), given by $\theta = 0, \pi$. In libration the value $\theta = \pi$ in general will not be

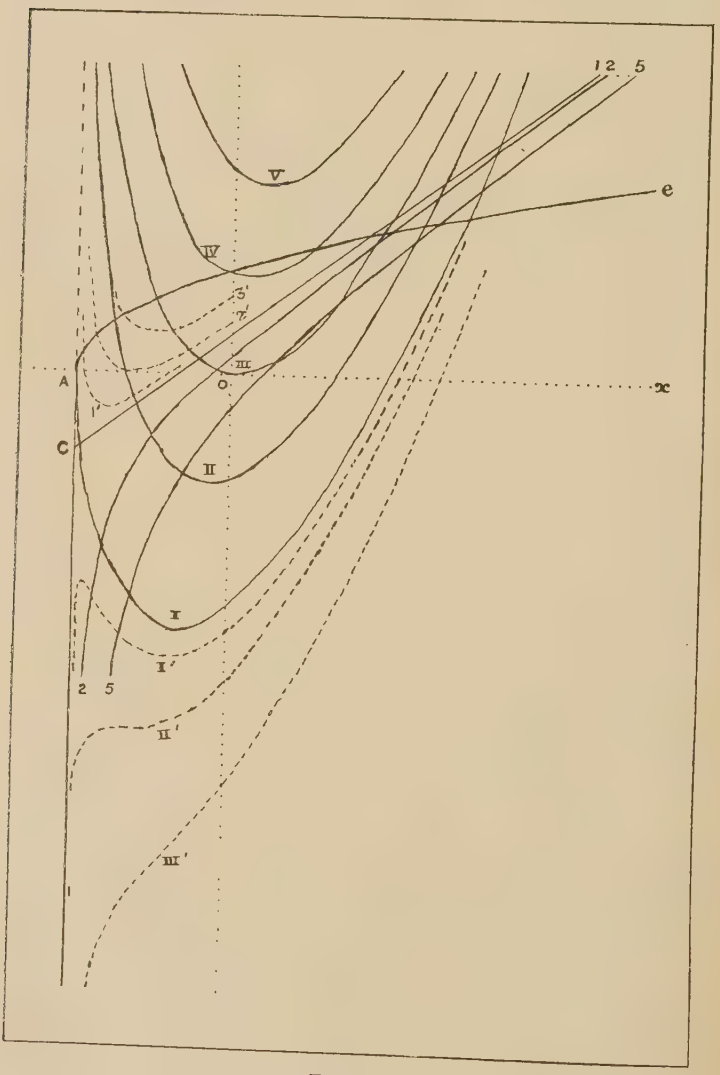


FIG. 1.

attained, so that $\theta = 0$ at both the maximum and the minimum. Also, x is at a maximum and minimum at the same places as e.

18. These facts are exhibited by the curves drawn to scale in fig. 1. All of them have x as abscissa.

The parabola Ae has e as ordinate and represents the equation $e^2 = E + \frac{2}{3}x$. Hence $oA = -\frac{3}{2}E$.

The hyperbolas 3′, 2′, 1′, 11, 22, . . . 55, represent (19) with y as ordinate, for the values

$$c = 4E^2,\ 3E^2,\ \tfrac{10}{4}E^2,\ \tfrac{9}{4}E^2,\ E^2,\ 0,\ -E^2,\ -2E^2,$$

respectively. In order to avoid complication in the figure, the hyperbolas 33, 44, lying between 11, 55 are omitted. The hyperbola 11 consists of the portions of the asymptotes shown.

The curves III′, II′, I′, I, II, III, IV, V show the corresponding relations between θ, x, the ordinates representing the function $6pE^{-\frac{2}{3}}\cos\theta$. They have the line $x = -\frac{3}{2}E$ as an asymptote, except in the case $c = \frac{9}{4}E^2$, when it becomes the tangent at A. The lowest point of each curve corresponds to $y = 0$, and it therefore has the same abscissa as the point of intersection of the corresponding hyperbola with the x-axis.

19. No ordinate of the curves III′. . . . V may numerically exceed $6pE^{-\frac{2}{3}}$. There is libration when E, c are such that this limit cannot be attained in the curve. When $c \leqq 0$ (III, IV, V) the only possible motion is a libration, but no motion will be possible if the ordinate of the lowest point of any curve for $c < 0$ is less than $6pE^{-\frac{2}{3}}$. If $c > 0$, there will be libration of the numerical value of the ordinate if the lowest point is less than $6pE^{-\frac{2}{3}}$. If the latter condition is not fulfilled, there will be two modes of motion in which θ makes complete revolutions, and each of these is possible for the same pair of values of c, E. In one of them, x oscillates about a negative value, in the other about a positive value.

An interesting deduction can be drawn from the last result, in which $\frac{9}{4}E^2 > c > 0$ and E is such that θ is not librating. Suppose that an external mass enters the solar system and passes near the asteroid when θ is nearly equal to π. It may cause θ, x to change in such a way that when the mass has left the neighbourhood of the asteroid the latter will be found on the other portion of the curve without any permanent change in c, E. It will then have crossed the libration region and afterwards move permanently with a real mean motion less instead of greater (or greater instead of less) than twice that of Jupiter.

20. The curves 3′, 2′, 1′, and III′, II′, I′ (broken lines) are for $c > \frac{9}{4}E^2$. The hyperbolas lie above the oblique asymptote, but there can be no libration unless they intersect the x-axis, that is, unless $c < 3E^2$. If the latter relation holds, the only possible

librations are those in which $-pE^{\frac{2}{3}}$ lies between the maximum and minimum shown, for example, in the curve I′. This libration takes place about the value π of θ. Such a motion, even if it be possible with the actual existence of short-period terms, must be rather unstable. If $pE^{-\frac{2}{3}}$ is numerically greater than the ordinate of the minimum, the motion is such that θ, in the interval between $\frac{1}{2}\pi$, π, reaches a maximum, diminishes to a value greater than $\frac{\pi}{2}$, and afterwards increases to π. A similar series of changes in the interval between π, $\frac{3}{2}\pi$, takes place in the reverse order. This curious part of the motion takes place with x negative. As long as x is positive, the changes of θ are normal. The existence of the two values of e_0 in (iii) of No. 17 is thus explained.

21. The figure has been drawn for E positive. When E is negative, A is to the right of O, and C is in the first quadrant. Hence x is always positive. The hyperbolas 1′, 2′, 3′ no longer intersect the x-axis, and the curves I′, II′, III′ have no maxima or minima in the finite part of the plane. Libration is therefore only possible when $c < \frac{9}{4}E^2$. It is evident that the range of values of E in libration is much smaller than when E is positive.

22. The circumstances of any case of motion can be read off from a figure when the numerical values of c, E are known. Take $OA = -\frac{3}{2}E$. Draw the hyperbola (19), the lines $y = \pm 6pE^{-\frac{2}{3}}$ and the curve whose equation is

$$y\sqrt{E + \frac{2}{3}x} = (x^2 - c)E^{-\frac{4}{3}}. \quad . \quad . \quad (X)$$

If E is negative, replace E by $-E$ in $E^{-\frac{4}{3}}$. In the cases where more than one kind of motion is possible, one value of x must be known in order to determine the portion of the curve (X) in which motion is taking place; as c, E are determined from the observation of at least one set of values of x, e, θ, such a value is always known.

23. *The limits of the eccentricity in cases of libration.*—I suppose that $c < \frac{9}{4}E^2$. The maxima and minima of e occur when $\theta = 0$. Inserting this value in (18) and eliminating x, y, we obtain

$$e^4 - 2e^2E - \frac{8}{3}pe + E^2 - \frac{4}{9}c = 0.$$

This equation has only two real positive roots if $E^2 > \frac{4}{9}c$; one is the maximum, and the other the minimum value of e.

The corresponding values of x are given by $2x = 3(e^2 - E)$,

and those of $y(=\dot{\theta}/2n')$ by $y=x-p/e$. The last equation shows that x cannot be always negative, for then y would be always negative, and therefore θ always diminishing. Hence x is always positive at its maximum; it may be positive or negative at the minimum.

Let the maximum be denoted by the suffix 2, and the minimum by the suffix 1. At the former, E_s can have its greatest value; at the latter it may be zero. Since x_2 is positive, we have from (18),

$$\left.\begin{aligned} e_2^2 &= E + E_s + \frac{2}{3}\sqrt{c+6pe_2}, \\ e_1^2 &= E \quad \pm \frac{2}{3}\sqrt{c+6pe_1}, \end{aligned}\right\} \quad . \quad . \quad (23)$$

Hence

$$e_2^2 - e_1^2 = E_s + \frac{2}{3}\sqrt{c+6pe_2} \mp \frac{2}{3}\sqrt{c+6pe_1}. \quad . \quad (24)$$

Considering e_1, e_2 as the co-ordinates of a point, I have traced the curves (24) for different values of c with $E_s = \cdot 003$, $p = \cdot 006$. They have to be limited in two ways: first, by the condition $E^2 > \frac{4}{9}c$; and, second, by the condition

$$p > \left| \left\{ -E + \sqrt{E^2 - \frac{1}{3}c} \right\} \left\{ \frac{1}{3}E + \frac{2}{3}\sqrt{E^2 - \frac{1}{3}c} \right\}^{\frac{1}{2}} \right|, \quad (25)$$

derived from (22). Both conditions must hold when we replace E by $E+E_s$. It is easily seen that the right-hand member of (25) decreases with this substitution, so that (25) was used for one limit and $(E+E_s)^2 > \frac{4}{9}c$ for the other. It appears that (25) is equivalent to

$$e_1 < e_0 = \left\{ \frac{1}{3}E + \frac{2}{3}\sqrt{E^2 - \frac{1}{3}c} \right\}^{\frac{1}{2}}. \quad . \quad (26)$$

The discovery of the limiting curves could therefore be made without tracing the curves (24), in the following way:—

Eliminating c by means of $(E+E_s)^2 = \frac{4}{9}c$ from (25), we obtain

$$2E + 2E_s = e_1^2 + E_s - \frac{8}{3}p\frac{e_1}{e_1^2 + E_s}, \quad \frac{8}{3}p = e_2^3 - e_2(2E + 2E_s),$$

which is traced by giving a value to e_1 and finding by trial the corresponding value of e_2.

Again, eliminating c, E between (25), (26), after putting $e_1 = e_0$ in the latter, we obtain

$$A\left(A + \frac{4}{3}\frac{p}{e_1}\right) = \frac{8}{3}p(e_2 - e_1), \quad A = e_2^2 - e_1^2 - E_s,$$

which is traced in a similar way.

The following table shows points on these two curves with $p = \cdot 000716$, for $E_s = \cdot 003$ and $E = \cdot 006$:—

$E_s = \cdot 003$ $(E + \cdot 003)^2 = \frac{4}{9} c.$		$E_s = \cdot 003$ $e_1 = e_0.$		$E_s = \cdot 006$ $(E + \cdot 006)^2 = \frac{4}{9} c.$		$E_s = \cdot 006$ $e_1 = e_0.$	
e_1	e_2	e_1	e_2	e_1	e_2	e_1	e_2
·000	·132	·020	·074	·000	·140	·020	·096
·010	·116	·030	·082	·010	·132	·030	·104
·020	·103	·050	·087	·030	·120	·050	·117
·030	·096	·100	·131	·050	·117	·100	·148
·040	·093	·150	·172	·060	·118	·150	·185
·050	·093	·200	·217	...	...	·200	·226
...	...	...	...	...	...	·300	·316

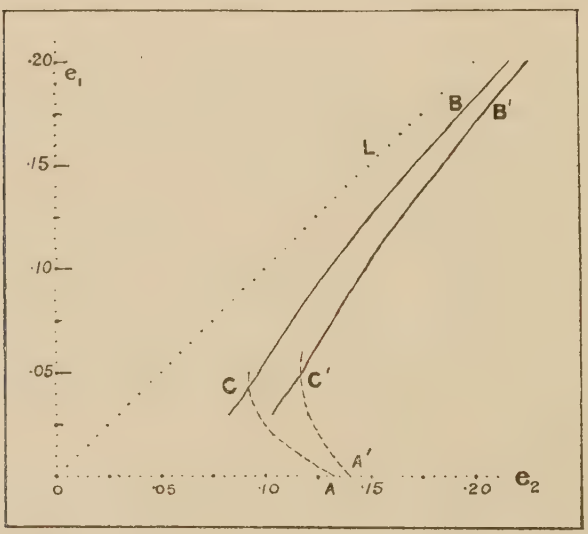


FIG. 2.

The four curves with co-ordinates e_2, e_1, are shown in fig. 2 : AC, CB corresponding to $E_s = \cdot 003$ and A'C', C'B' to $E_s = \cdot 006$. The region for which libration is possible must lie to the right of ACB in the former case and to the right of A'C'B' in the latter. The curves CB, C'B' have the line OL as asymptote. The points of intersection, C, C', give the least values of e_2.

Referring back to the table in No. 16, it appears that E_s should have a value near ·005, and from this we conclude that *in libration, whatever the initial eccentricity, its osculating value must at some time reach the value* ·11 *at least.*

24. *Effect of the eccentricity of Jupiter.*—We must add to R_c the term $p'e' \cos \theta'$, where

$$\theta' = 2n' \int x dt + \epsilon_1 - 2\epsilon' + \varpi' ,$$

and p' has the factor m' and is a function of $a_0/a' = 4^{-\frac{2}{3}}$. Since, with the previous limitations, we may neglect $d\epsilon/dt$, we have the additions $-6p'e'\cos\theta'$, $-4n'p'e'\int\sin\theta' dt$ to the right-hand members of the first and second of equations (18), and the additional equation $d\theta'/dt = 2n'x$.

It is possible to integrate $\sin\theta'$ as follows. We have

$$\frac{de}{dt} = \frac{na(1-e^2)}{\mu e}\frac{\partial R}{\partial \epsilon} - \frac{na\sqrt{1-e^2}}{\mu e}\left(\frac{\partial R}{\partial \epsilon} + \frac{\partial R}{\partial \varpi}\right)$$
$$= -\frac{na}{\mu}\left(\frac{1}{2}e^2\frac{\partial R}{\partial \epsilon} + \frac{1}{e}\frac{\partial R}{\partial \varpi}\right) = -2n'p\sin\theta,$$

under the same limitations. Also

$$2n'p\cos\theta = ex - e\frac{d\theta}{dt} = e\left(\frac{d\theta'}{dt} - \frac{d\theta}{dt}\right). \qquad (27)$$

Using these results, we find that

$$\sin\theta' = \sin\theta\cos(\theta'-\theta) + \cos\theta\sin(\theta'-\theta)$$
$$= \frac{1}{2n'p}\left\{-\frac{de}{dt}\cos(\theta'-\theta) + e\left(\frac{d\theta}{dt} - \frac{d\theta'}{dt}\right)\sin(\theta'-\theta)\right\}$$
$$= -\frac{1}{2n'p}\frac{d}{dt}\left\{e\cos(\theta'-\theta)\right\}.$$

Hence the new form of equations (18) is

$$\left.\begin{aligned} x^2 &= c + 6pe\cos\theta - 6p'e'\cos\theta', \\ e^2 &= E + \frac{2}{3}x + 2\frac{p'}{p}ee'\cos(\theta-\theta'), \\ \frac{1}{2n'}\frac{d\theta}{dt} &= x - \frac{p\cos\theta}{e}, \qquad \frac{1}{2n'}\frac{d\theta'}{dt} = x. \end{aligned}\right\} \qquad (28)$$

We are dealing with cases of minimum libration of θ taking place about a value $\theta = 0$. If $|\theta| < \frac{1}{2}\pi$, the equation (27) shows that $\theta' - \theta$ is always increasing. Since θ librates, θ' cannot librate, and therefore $\cos\theta'$, $\cos(\theta'-\theta)$ must be capable of taking all values between ± 1. The last result is obviously true when θ librates through a larger angle, the mean value of $d\theta'/dt - d\theta/dt$ being positive. The values 0, π of θ' can be attained in general for any value of θ. It follows that the true mean value of x can only be zero in exceptional cases. In other words,

$$nt = 2n't + \beta t + \beta',$$

where β is a constant unequal to zero, and β' is a periodic function. This, of course, does not need the introduction of θ' for proof.

The result just obtained plays no part in an explanation of the absence of asteroids for which θ librates. It is the absence of zero *osculating* mean motions which needs an explanation. Further, in the cases considered here, β is quite small compared with β' and x passes through its zero value not very far from the time when $\theta = 0$

25. The maximum and minimum of e still take place at $\theta = 0$. Inserting this value in the first two of equations (28), we have

$$x^2 = c + 6p\left[e - \frac{p'}{p}e' \cos\theta'\right],$$

$$\left[e - \frac{p'}{p}e' \cos\theta'\right]^2 = \mathrm{E}' + \frac{2}{3}x, \quad \mathrm{E}' = \mathrm{E} + \left[\frac{p'}{p}e' \cos\theta'\right]^2.$$

The previous work is available if we replace e by $e - p'e' \cos\theta'/p$, and E by E′. Since the maxima and minima of $\cos\theta'$ are independent of θ, we must replace e_2 by $e_2 + \frac{p'}{p}e'$, e_1 by $e_1 - \frac{p'}{p}e'$.

The quantity E′ is the same at both limits, and as it is ultimately eliminated, the addition to E makes no difference in the results.

The maximum value of e' is ·06. Also $p' = ·000258$, $p = ·000716$. Hence $p'e'/p = ·0216$.

The least value of e_2 occurred with $e_1 = ·05$, so that the subtraction of ·02 from e_1 does not make the new e_1 negative. *The addition to e_2 makes the least value of the maximum eccentricity* ·13.

26. *Higher terms in the disturbing function.*—The next step should consist in taking account of the critical terms with arguments 2θ, $\theta - \theta'$, $2\theta'$ having characteristics e^2, ee', e'^2, respectively. Owing to the larger numerical coefficients, it would seem that they must have at least as much effect on the range of e as the term with characteristic e'. The additions to the right-hand member of the equation for x^2 are

$$-6[p_0e^2 + p_2e^2 \cos 2\theta - p_3ee' \cos(\theta - \theta') + p_4e'^2 \cos 2\theta],$$

where

$$p_0 = ·000233, \; p_2 = ·00102, \; p_3 = ·00288, \; p_4 = ·00216.$$

Compared with the value $p' = ·000258$, the last three of these are large. Unfortunately their maxima and minima are not independent of those of θ, and a much more extended investigation is necessary to obtain numerical results. Nevertheless it seems probable that they will have the effect of adding to the range of e. Further, most of the asteroid orbits have some inclination to that of Jupiter, and it is well known that the numerical parts (those depending on a_0/a') of the coefficients of the terms arising from this source are in general larger than of those depending on e^2. These, with perturbations from other sources, make it quite

probable that the least maximum that e can have if libration exists is not far short of ·20.*

The course of the argument shows that the neglected short-period terms in the third and fourth of equations (28) will not exercise much influence on the results; if anything, they should tend to increase the least range of e since they are effective chiefly at the limit $d\theta/dt = 0$.

As far as the limiting curve is concerned, the chief difficulties occur with small values of e_1. When e_1 exceeds ·15, the range of e is small compared with its mean value, and expansions in powers of $(e - e_0)/e_0$ become possible. The square root of e^2 in the second of equations (18) or (28) may then be extracted and the processes much simplified thereby.

27. *Other critical cases for characteristics of the first order.*—In all such cases we have

$$s = \frac{i+1}{i} \quad \text{or} \quad \frac{i}{i+1},$$

where i is an integer. If we multiply

$$x, \sqrt{c} \text{ by } \left[\frac{p^2}{q^2} \cdot \frac{1}{s-1}\right]^{\frac{1}{3}}; \; e, \sqrt{\mathrm{E}} \text{ by } \left[\frac{p}{q} \cdot \frac{1}{(s-1)^2}\right]^{\frac{1}{3}}; \; \frac{d\theta}{dt} \text{ by } \left[\frac{p}{q}(s-1)\right]^{\frac{2}{3}},$$

the equations (18) become

$$x^2 = c + 6qe\cos\theta, \; e^2 = \mathrm{E} + \frac{2}{3}(s-1)x, \; \frac{1}{2n'}\frac{d\theta}{dt} = \frac{x}{s-1} - \frac{q}{e}\cos\theta,$$

which are the equations for any s and any coefficient q. In dealing with the maximum and minimum of e, it is therefore only necessary to change the scale of the curves in fig. 2, to obtain the proper results for these cases. It is true that the value of θ must be changed when $d\theta/dt$ is changed, but when $\theta = 0$ or $d\theta/dt = 0$—the only values used in the discussion—the change makes no difference. It must be noticed that E_s will have been multiplied by the same factor as E. Really, E_s does change, but not in the proportion of this factor. However, if we have previously calculated a few of the curves in fig. 2 for different values for E_s, we can easily select the one suitable to the case in question.

Application to the Asteroids of the Type 2/1.

28. It has been stated that the formulæ are applicable to cases in which s is nearly equal to 2 even when there is no libration. From the observed osculating elements n, ϵ, ϖ, we can obtain x, θ, c, E from equations (28). Tests can then be made with the conditions necessary for libration, namely,

$$\mathrm{E}^2 - \tfrac{1}{3}c > 0, \quad |\, e_0 x_0 \,| < p,$$

* This result was stated in an address before the Amer. Assoc. Adv. Sc. See *Science* for 1911 January 20.

where

$$e_0 = \left\{ \frac{1}{3}\mathrm{E} + \frac{2}{3}\sqrt{\mathrm{E}^2 - \frac{1}{3}c} \right\}^{\frac{1}{2}}, \quad x_0 = -\mathrm{E} + \sqrt{\mathrm{E}^2 - \frac{1}{3}c}\,.$$

When E is negative the value for e_0 implies the further condition $\mathrm{E}^2 > \frac{4}{9}c$. The table which follows gives approximate results for five asteroids whose mean motions are less than $2n'$, and eight whose mean motions are greater than $2n'$.

Asteroids of Type 2/1.

No.	Period.	e.	ϵ.	ϖ.	θ.	θ'.	10^2x.	10^4c.	$10^2\mathrm{E}$.	$10^4(\mathrm{E}^2-\frac{1}{3}c)$.	e_0.	10^2x_0.	$10^3e_0x_0$.
	d		°	°	°	°							
300	2100	·043	129	325	91	139	3·14	9·3	− 2·01	+ ·93	imag.		
325	2101	·159	184	60	241	194	3·09	12·1	+ ·09	− 4·03	,,		
108	2101	·106	216	166	19	226	3·09	4·7	− ·61	− 1·20	,,		
318	2104	·063	43	76	116	53	2·95	10·3	− 1·67	− ·66	,,		
581	2104	·044	343	64	44	353	2·95	8·1	− 1·88	+ ·84	,,		
122	2105	·054	130	191	318	140	2·90	6·1	− 1·45	+ ·06	,,		
530	2118	·181	221	318	176	231	2·27	12·4	+ 1·41	− 2·16	,,		
175	2120	·192	202	328	167	212	2·17	12·1	+ 1·77	− 0·91	,,		
Jup.	4332	·048	182	13	...	..	...	...	...	...	...	[10^3p = ·72]	
401	2222	·047	334	236	207	344	−2·52	8·9	− 1·78	+ ·21	imag.		
525	2229	·371	98	47	142	108	−2·83	20·3	+14·58	+205·8	·380	− ·23	− ·8
643	2244	·077	145	90	232	155	−3·48	13·5	+ 2·85	+ 3·63	·149	− ·94	− 1·4
566	2245	·121	166	25	188	176	−3·52	16·8	+ 3·40	+ 5 96	·166	− ·96	− 1·5
466	2247	·083	292	195	124	302	−3·61	15·4	+ 3·39	+ 6·37	·168	− ·87	− 1·4

There is only one asteroid which gives a doubtful result, that numbered 525, and this doubt arises from its large eccentricity. The inclusion of the terms depending on the squares and higher powers of the eccentricities will probably modify the values of c, E to a considerable extent, so that the ratio $|e_0x_0/p|$ may fall below unity. It is not my purpose to examine this question in detail.

Yale University:
1912 *February* 12.

Radiant Points of Shooting Stars observed at Bristol, chiefly in 1899–1911. By W. F. Denning.

The General Catalogue of Radiant Points published in 1899, and forming part of vol. liii. of the *Memoirs*, contained all the showers observed here prior to the year mentioned. Since that period my observations of shooting stars have not been so numerous as during previous years, as I was much engaged in telescopic studies on the planets until 1906, when ill-health necessitated prolonged rest from that work; but during the past six years meteoric observations have been continued. In and since 1899 I have seen about 4770 shooting stars, and of these I registered the flights of 3090. The 240 radiant points here given are the best positions I have found, and they are based upon about 2570 meteors. I believe they may nearly all be relied on as genuine positions, every care having been taken to avoid centres depending upon the mere chance intersection of meteor-tracks. Some of the positions rest upon few meteors; but probably they are little less accurate on that account, for I think that four, five or six well-observed paths will indicate a radiant point as well as a great number, provided the observer is sure (from long experience and a due regard for all the details which alone can assist his judgment) that he has apportioned the meteors to their proper centres. Probably the average error of the positions in the table is within 2°.

In addition to the showers here represented, a great number of others were more feebly and less certainly manifested. In fact, the majority of the streams visible in our skies are extremely feeble, and one observer cannot easily ascertain them unless his watchings are very extensive and prolonged through the whole night. Even then he must combine the results of several nights in order to acquire sufficient materials to denote the weaker radiants. I have sometimes watched the sky for thirty or forty hours within a week and recorded 300 or 400 meteors, and even then have only secured just sufficient evidence, from 4 or 5 meteors, of the more attenuated systems visible at such periods.

With regard to the fact of stationary radiation, to the displacement in the Perseid radiant, and to certain other features mentioned in previous papers, I can say only that recent observations here have served to substantiate my conclusions.

In order to derive the centres more satisfactorily I have in some cases utilised the data obtained in several years. This seems very desirable in instances where nearly exhausted streams have to be dealt with; for the collected materials for similar periods in several years will often corroborate each other and indicate accurate radiants.

With reference to the accordance of cometary and meteoric

orbits, I have met with very few, if any, suggestive instances of identity beyond the well-known ones. Relatively very few comets pass sufficiently near to the earth to provide us with visible meteoric showers.

Professor A. S. Herschel gave a valuable list of the radiant points of comets for both the North and South hemisphere in the *British Association Report on Luminous Meteors for* 1874, and a few observed meteoric displays agree tolerably well with them in position and date, but they may quite possibly be chance coincidences without any physical association whatever.

Abbreviations in sixth column.

R. Rapid.	T. Trains.
S. Slow.	L. Long.
K. Streaks.	W. White.
B. Bright.	V. Very.

Ref. No.	Date of Observation.	Radiant Point. α δ	No. of Meteors.	Apparent Velocity, etc.	No. in General Catalogue.
		° °			
1	1902 July 9–10	0+44	4	R	8
2	1901 July 18–24	2+43½	5	R	8
3	1911 Aug. 30–Sept. 1	2− 2	6	SBLT	2
4	1910 Sept. 24–27	4+28	6	VS	16
5	1908 Aug. 3–7	5+44	6	RK	8
6	1900 Sept. 23–29	6+11	6	S	3
7	1902 Aug. 26–Sept. 7	11+23	4	R	?16
8	1911 July 20–26	11+50	6	RK	44
9	1911 Sept. 12–24	13+ 5	6	SLT	11
10	1901 Aug. 16–19	14+ 7		RK	11
11	1899 Aug. 9–11	16+31	4	RK	13
12	1902–1905 July 5–13	22+22	7	RK	14
13	1900 Sept. 19–Oct. 1	23+57	5	RW	15
14	1877–1900 Sept. 14–28	23+42	12	R	25
15	1901–04 July 15–21	23+44	7	RK	25
16	1911 July 16–27	23+42	11	RK	25
17	1901 Aug. 15–21	24+43	5	RK	25
18	1900 July 15–24	24+43	7	RK	25
19	1900 Sept. 19–Oct. 1	27+ 3	6	RW	...
20	1885 Sept. 3–17	29+36	6	R	25
21	1902 Sept. 3–7	30+37	8	R short	25
22	1885–1901 July 9–21	30+36	7	RBK	25
23	1886–1902 Aug. 27–Sept. 9	33+54	8	RK	30

Ref. No.	Date of Observation.	Radiant Point. α δ	No. of Meteors.	Apparent Velocity, etc.	No. in General Catalogue.
24	1901 Aug. 10–21	33°+18°	4	RK	27
25	1903 Oct. 22–27	40+21	4	S	38
26	1899 Aug. 11	40+21	5	RK	38
27	1901 Aug. 10–21	42+12	4	RK	36
28	1902 Sept. 3–7	42+56	4	RK	39
29	1900 Oct. 23–27	43+12	8	S	36
30	1904 Nov. 14	43+21	4	S	38
31	1908 July 26	45+85	4	S	52
32	1900 Oct. 23–27	45+27	8	R	42
33	1900 Aug. 24–Sept. 2	46+43	5	RK	43
34	1899 Aug. 13–17	47+44	6	RK	43
35	1900–11 Sept. 29–Oct. 5	47+43	14	R	43
36	1900 Oct. 23–27	47+43	6	S	43
37	1901 Aug. 20–22	47+43	4	RK	43
38	1877–1901 Aug.–Sept.	47−11	5	RBL	41
39	1906 July 24–Aug. 14	47+43	7	RK	43
40	1911 Aug. 14–18	48+43	6	RK	43
41	1901 Aug. 18–21	57+ 9	4	RK	49
42	1900 Oct. 23	57+ 9	5	RBW	49
43	1900 Nov. 13	58+10	6	S	49
44	1911 Feb. 19–26	58+72	6	VS	45
45	1911 Sept. 17–20	60+50	4	RK	56
46	1901 Aug. 10–19	63+30	6	RW	58
47	1911 Aug. 28–30	65+22	4	RWK	53
48	1900 Aug. 22–Sept. 2	71+65	6	R	62
49	1901 Aug. 10–19	71+52	4	RK	63
50	1900 Sept. 23–30	72+65	5	R	62
51	1903 Oct. 23–24	74+14	4	R	64
52	1902 Aug. 24–Sept. 7	74+41	7	VRK	65
53	1876–1903 Feb. 7–23	75+41	7	SBT	65
54	1900 Sept. 23–27	76+32	5	RK	67
55	1900 Aug. 16–Sept. 2	89+42	5	RK	72
56	1900 Oct. 26–27	91+16	5	RK	77
57	1900–03 Oct. 22–23	91+59	7	R	76
58	1900 Oct. 23	92+15	5	RK	77
59	1903 Oct. 22	93+16	10	RK	77
60	1902 Sept. 29–Oct. 2	95+71	4	R	82
61	1900 Oct. 23	99+13	7	RK	79

Ref. No.	Date of Observation.		Radiant Point. α δ	No. of Meteors.	Apparent Velocity, etc.	No. in General Catalogue.
62	1901	Oct. 20	99°+13°	4	RK	79
63	1903	Oct. 22	99+13	10	RK	79
64	1877–1902	Sept. 7–25	100+13	10	VRK	79
65	1901	Aug. 10–20	106+52	5	RK	84
66	1903	Jan. 2	116+29	4	Slowish	94
67	1900–03	Oct. 22–23	117+30	5	RK	94
68	1900–03	Oct. 22–27	117+46	6	RW	103
69	1877–1902	Sept. 4–21	118+31	7	RLK	94
70	1902	Sept. 3–7	118+83½	9	R	92*a*
71	1900	Oct. 26–27	123+42	4	R	97
72	1900–03	Oct. 22–27	128+33	5	RKW	102
73	1903	Oct. 22–24	133+77	5	R	103
74	1877–1903	Jan. 17–25	143+38	9	RW	? 107
75	1904	Nov. 14	144+37	3	RK	? 107
76	1904	Nov. 14	150+23	33	RK	115
77	1900	Nov. 13–14	151+22	4	RK	115
78	1903	Nov. 15	151+22	237	RK	115
79	1911	Feb. 19–Mar. 1	155+14	7	ST	120
80	1877–1903	Jan. 22–Feb. 1	159+27	5	S	121
81	1910	Nov. 5–7	163+57	5	RK	123
81*a*	1877–87	Oct. 14–17	167+32½	6	RK	125
82	1903–11	May	195+57	7	SB	146
83	1901	April 18–21	204+57	4	R	155
84	1903	Jan. 3–27	220+13	5	RK	...
85	1911	Jan. 2–3	227+52	7	RL	173
86	1903	Jan. 3	228+53	7	R	173
87	1901	April 18–21	233+34	5	R	176
88	1901	April 18–24	240+46	6	RF	181
89	1910–11	Sept. 1 and 24–7	242+70	6	S	...
90	1887	June 13–25	244−10	5	SL	...
91	1903	June 22–July 1	245+64	5	S	180
92	1911	May 2–31	246+29	7	RW	...
93	1911	May	247+4	5	RW	187
94	1903	May 18–26	248+29	6	S	...
95	1902	July 10–13	254+47	4	S	...
96	1911	May 18–20	255+73	5	R	186
97	1902	Sept. 25–29	255+46	5	R and S	...
98	1886–1903	May 25–31	255+37	5	S	195

Ref. No.	Date of Observation.		Radiant Point. α δ	No. of Meteors.	Apparent Velocity, etc.	No. in General Catalogue.
99	1911	May 24–29	260°+61°	5	S	198
100	1901	April 20–21	261+36	8	RBW	202
101	1901	Aug. 11–20	263+62	7	S	198
102	1886–1903	May 20–26	263+38	5	R	202
103	1901	April 18	267+33	3	RK	209
104	1908	June 21–26	268+25	5	R	205
105	1901	Aug. 10–15	269+47	7	S	208
106	1901	April 21	270+33	25	RK	209
107	1908–11	July 31–Aug. 7	270+48	15	S	208
108	1911	July 1–2	270+30	5	S	?206
109	1899	July 4–14	272+68	5	RW	213
110	1886–1903	May 25–27	273+22	5	R	205
111	1911	July 24–26	273− 3	5	VS	211
112	1899	July 30–Aug. 17	277+70	15	SW	213
113	1908	July 19–Aug. 1	280+57	11	S	215
114	1886–1903	May 25–June 4	280+31	10	R	216
115	1901	April 21–24	288+53	5	RF	229
116	1901	Aug. 10–21	290+53	10	S	229
117	1901	July 19–24	290+24	8	S	222
118	1900	July 18–30	291+52	8	S	229
119	1900	July 24–30	291+59	6	S	224
120	1899	Aug. 16–18	291+60	5	S	224
121	1911	Aug. 18–22	293+65	6	S	...
122	1911	Aug. 18	293+51	6	S	229
123	1899	Aug. 12–16	293+60	8	S	224
124	1908	July 21–31	298−15	8	SL	237
125	1902	July 6–9	299+24	6	S	232
126	1900–01	July 18–24	300+23	5	R	232
126*a*	1911	July 22–27	302+21	7	R	232
127	1905–08	July 19–Aug. 5	302+22	15	S	232
128	1908	June 26–28	302+22	4	R	232
129	1900–11	Aug. 22–Sept. 3	304+23	8	S	232
130	1911	Aug. 12–25	304+24	17	R	232
131	1901	July 15–24	304− 8	6	S	237
132	1900	July 15–30	305−12	8	S	237
133	1911	July 27–Aug. 1	308−12	11	VSL	237
133*a*	1911	Aug. 28, etc.	308+61	4	R	246
134	1911	Sept. 18–24	309+61	6	RW	246

Ref. No.	Date of Observation.		Radiant Point. α δ	No. of Meteors.	Apparent Velocity, etc.	No. in General Catalogue.
135	1908	Aug. 27–29	310°+60°	4	RW	246
136	1910	Nov. 5–9	310+47	5	R	245
137	1901	Aug. 10–22	310+61	11	S	246
138	1901	July 15–21	311+13	5	S	263
139	1901	Aug. 11–18	312+13	5	S	263
140	1911	July 9–12	312+22	5	R	247
141	1898	July 18–28	314+48	6	R	245
142	1899	July 30–Aug. 14	315+77	10	S	240
143	1887	June 12–28	315+48	6	R small	245
144	1911	July 31, etc.	315+60	5	RW	246
145	1900	July 18–30	316+48	9	S	245
146	1876–96	Nov. 25–Dec. 12	316+60	13	S	246
147	1903–10	May 22–31 July 16–18 Oct. 7 Nov. 7–9	316+80	12	ST	240
148	1901	July 15–20	317+31	5	S	248
149	1911	July 8–15	318+31	6	RW	248
150	1908–10	July 28–Aug. 10	318+ 9	13	S	250
151	1908	Sept. 28–Oct. 9	320+40	6	S	...
152	1911	Aug. 18–25	320+11	4	VS	250
153	1902	July 10–13	323+22	5	S	...
154	1899	Aug. 14–17	323+23	7	SW	...
155	1900–02	Aug. 20–Sept. 18	323−24	6	VS	...
156	1901	Aug. 18–21	324+65	7	RW	?254
157	1877–85	July 1–5	329+72	6	R	254
158	1908	July 26–Aug. 6	331+70	4	R	254
159	1902–11	Sept. 24–29	331+58	6	S	256
160	1911	July 26–28	331+62	5	R	256
161	1886–1903	May 22–30	332+71	5	R	254
161*a*	1902–11	Sept. 24–29	332+58	6	S	256
162	1899	Aug. 9–12	332+27	8	S	259
163	1902	July 11–12	332+36	4	S	253
164	1900	Sept. 18–30	332+56	6	ST	256
165	1900	Aug. 22–26	333+28	11	R	259
166	1902	July 6–13	333+57	6	S	256
167	1900	Sept. 1–2	334+57	6	S	256
168	1908	Aug. 29, etc.	334+50	4	R	257

Ref. No.	Date of Observation.		Radiant Point. α δ	No. of Meteors.	Apparent Velocity, etc.	No. in General Catalogue.
169	1901	Aug. 10–22	335 + 1	15	S small	258
170	1911	July 22–27	335 + 51	10	RK	257
171	1900	Aug. 20–26	335 + 71	7	S	254
172	1901	Aug. 10–20	335 + 73	13	RW	254
173	1900	July 16–24	335 + 73	5	S	254
174	1908	July 28–Aug. 6	337 + 45	5	R	...
175	1902	Aug. 26–Sept. 7	337 + 82	10	VR	...
176	1908	July 26–Aug. 3	337 − 10	14	SLB	263
177	1910	July 30–Aug. 9	337 − 12	4	SL	263
178	1900	July 24–30	338 − 10	24	SL	263
179	1899	July 30–Aug. 13	339 − 11	8	SL	263
180	1901–02	July 7–15	340 + 41	5	RW	266
181	1909	Aug. 7–8	341 − 12	4	SL	263
182	1909	Sept. 3–7	342 + 42	5	R	266
183	1904	July 11–18	343 + 39	4	VR	266
184	1904	July 11	345 + 54	3	R	?270
185	1899	July 30–Aug. 16	345 ± 0	9	R	268
186	1901	Aug. 15–21	345 ± 0	7	R	268
187	1900	Aug. 18–26	347 + 1	12	R	268
188	1877–02	Sept. 6–Oct. 4	347 + 3	13	VS	268
189	1903	July 20–26	348 + 25	8	R	...
190	1901	Aug. 15–19	349 + 48	5	S	?270
191	1903	July 24–26	350 + 62	5	R	278
192	1901	Aug. 10–21	350 − 13	10	S	...
193	1902	Sept. 4–7	350 + 40	6	VR	271
194	1903	Sept. 1–7	351 + 56	5	S	270
195	1911	Aug. 30–Sept. 4	352 + 4	5	S	273
196	1908	Aug. 19–29	352 + 30	4	R	...
197	1901	Aug. 10–21	353 + 31	6	S	...
198	1885–1902	Aug. 25–Sept. 16	353 − 11	21	SL	...
199	1908–11	May 29–June 1 Aug. 6–7	353 + 39	7	VRK	271
200	1911	Aug. 30–Sept. 4	353 + 39	6	VRK	271
201	1908	Aug. 28–30	354 + 72	7	RW?	277
202	1908	July 19–31	355 + 73	4	RW?	277
203	1900	Aug. 22–26	356 + 60	5	S	278
204	1901	Aug. 21	358 + 18	5	R	272

Great Perseid Shower (arranged according to Date).

Ref. No.	Date of Observation.	Radiant Point. α	Radiant Point. δ	Meteors.
205	1877–1902 July 7–9	10°	+45½°	6
206	1901 July 21	23	+52	4
207	1908 July 26	25	+53	4
208	1903 July 26	26	+54½	3
209	1908 July 28–29	32	+52	8
210	1900 July 30	31	+54	10
211	1899 July 29–Aug. 2	34	+54	8
212	1911 July 31	34	+54	15
213	1908 Aug. 2	39	+56	6
214	1899 Aug. 6	40	+55	6
215	1908 Aug. 6	40	+57	6
216	1908 Aug. 7	41	+56	5
217	1909 Aug. 8	41	+57	6
218	1911 Aug. 11	39	+57	10
219	1899 Aug. 9	44	+57	26
220	1904 Aug. 9	42	+55	?
221	1899 Aug. 10	44	+57	72
222	1901 Aug. 10	44	+58	55
223	1910 Aug. 10	44	+56	46
224	1902 Aug. 10	45	+58½	15
225	1909 Aug. 10	43	+56	12
226	1907 Aug. 10	44	+56	58
227	1908 Aug. 10	43	+57	15
228	1899 Aug. 11	46	+57	69
229	1901 Aug. 11	45	+58	49
230	1907 Aug. 11	45	+57	30
231	1909 Aug. 11	46	+58	68
232	1904 Aug. 11	46	+58	37
233	1899 Aug. 12	48	+57	43
234	1902 Aug. 12	47	+58½	9
235	1903 Aug. 12	46	+58	20
236	1904 Aug. 12	47	+58	10
237	1907 Aug. 12	47	+57	68
238	1909 Aug. 12	47	+58	55
239	1899 Aug. 13	49	+58	10
240	1899 Aug. 14	50	+56	12
241	1911 Aug. 14	50	+59	12

Great Perseid Shower—continued.

Ref. No.	Date of Observation.		Radiant Point. α	δ	Meteors.
242	1901	Aug. 15	51°	+58°	6
243	1899	Aug. 16	53	+57	7
244	1900	Aug. 16	54	+58	5
245	1901	Aug. 16	53	+58	5
246	1911	Aug. 18	54	+58	8
247	1901	Aug. 18	55	+59	8
248	1901	Aug. 20	57	+58½	7
249	1901	Aug 21	61	+59	8
250	1900	Aug. 22	59	+59	5

The Perseids are No. 40 in *The General Catalogue of Radiant Points*, and are very swift meteors, leaving streaks. The first position I give for them, viz. July 7 to 9, 10° + 45½°, must be regarded as rather doubtful, pending further observation, as this radiant may really represent a shower of Andromedids. I cannot be absolutely certain that the Perseids begin to fall before the middle of July. Probably the shower is extended to August 25.

Bristol:
1912 *April* 22.

Correction.

The Secretaries are indebted to Professor T. J. J. See for a correction to the statement in *Monthly Notices*, 1912 February, p. 313, in which are attributed to him some views on the origin of binary stars which he has not entertained during the last four years. He no longer believes that bodies are thrown off or produced by the disruption of rotating nebulæ (vide *A.N.*, 4341-42).

Erratum in Annual Report, vol. lxxii.

Page 335, line 14, *for* α Ursæ Majoris *read* α Ursæ Minoris.